PY1E04: Lecture 6

Introduction to Physics
(Electromagnetism)

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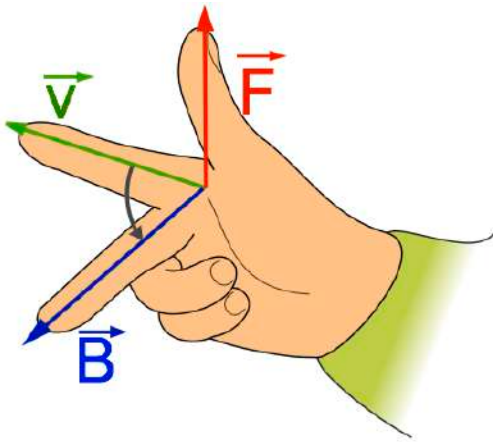
Reminder

$$\boldsymbol{F} = \boldsymbol{q}\ \boldsymbol{v}\ \boldsymbol{B} \sin\theta$$

where B is the magnetic field measured in Tesla

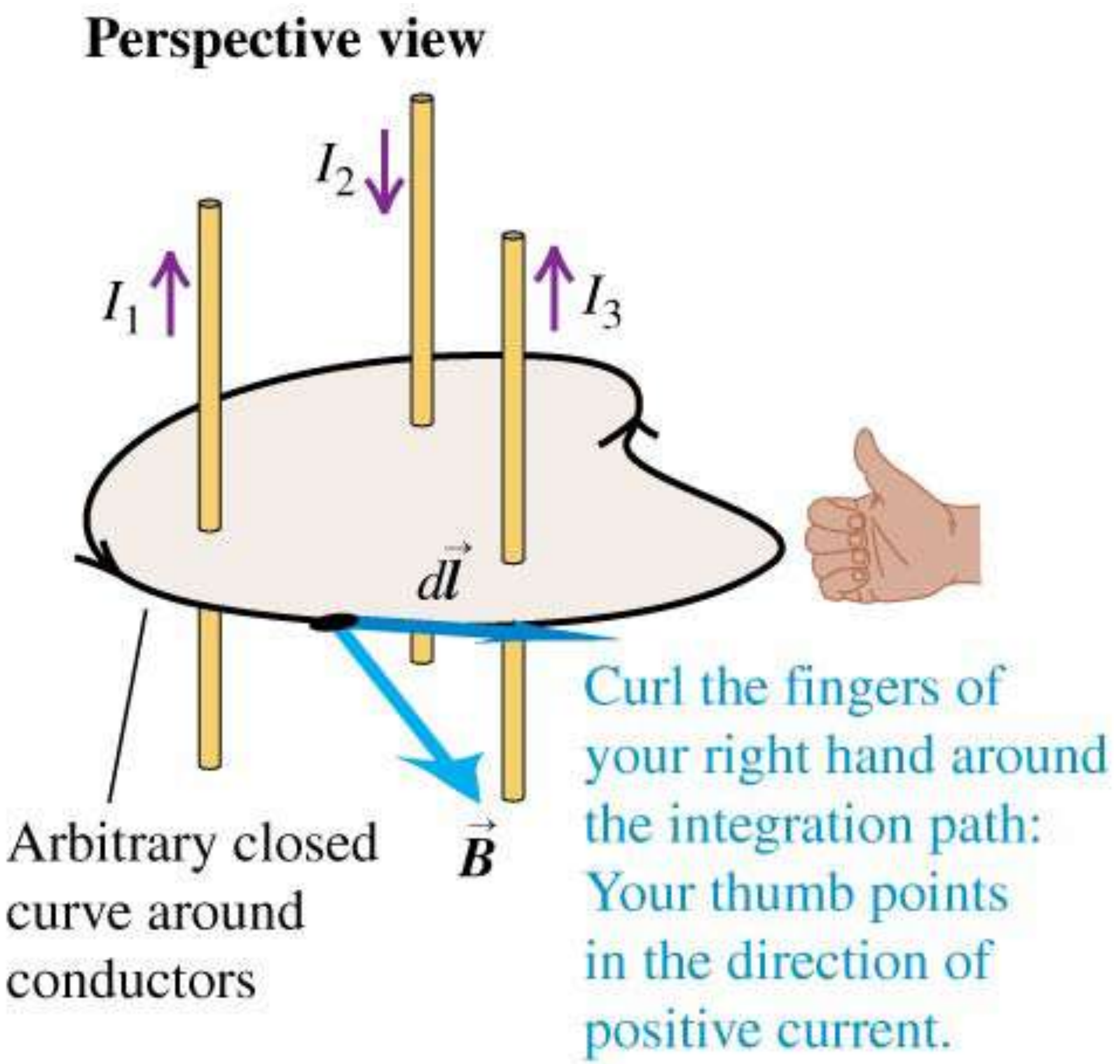


Gauss's Law for magnetism:

$$\oint \boldsymbol{B}.\,d\boldsymbol{A} = 0$$

The magnetic flux through any closed surface is zero.

Current carrying conductors

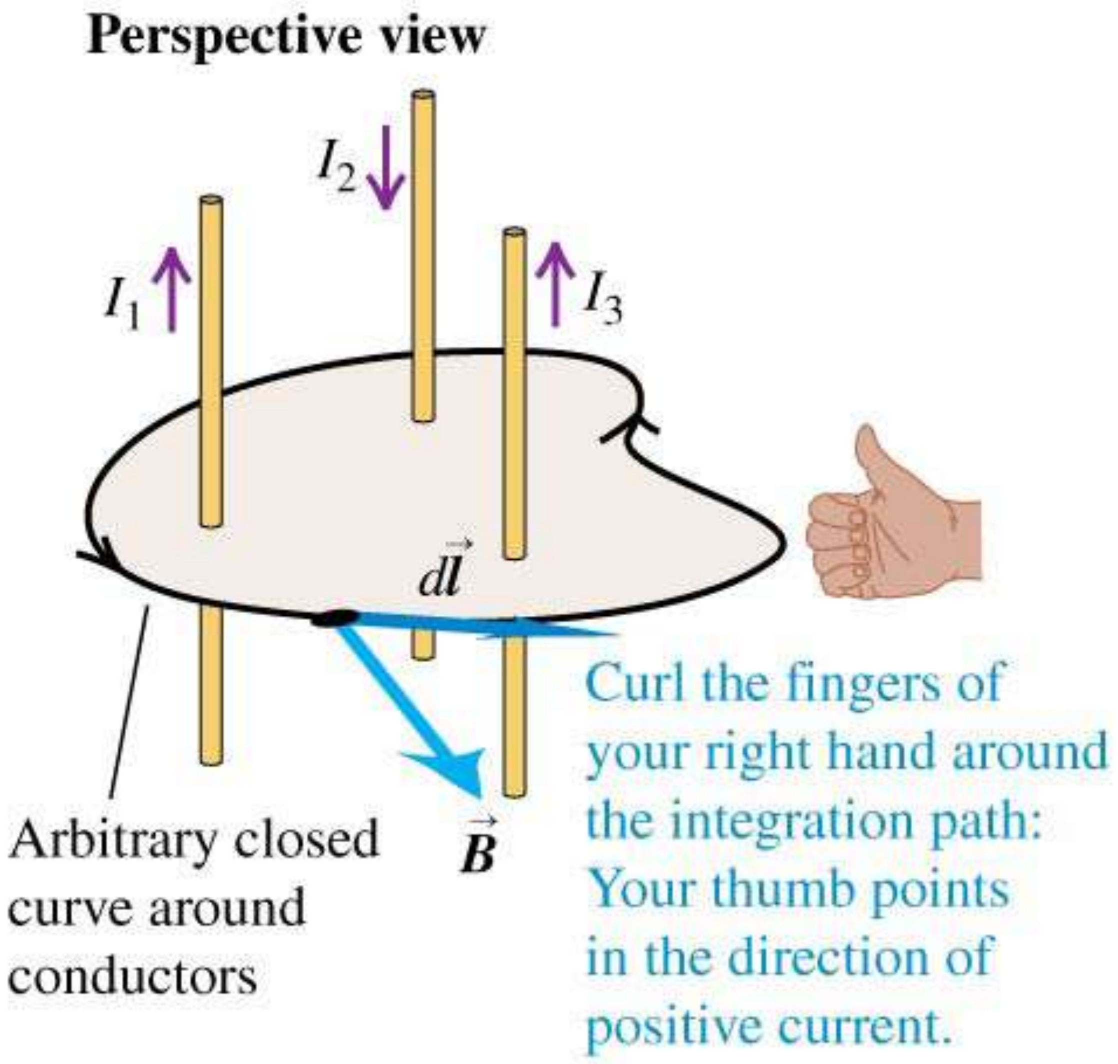


Ampere’s Law

Ampere’s law relates electric current to the line integral around a closed path.

Suppose several long, straight conductors pass through the surface bounded by the integration path.

Thus the line integral of the total magnetic field is proportional to the algebraic sum of the currents.



Ampere’s Law

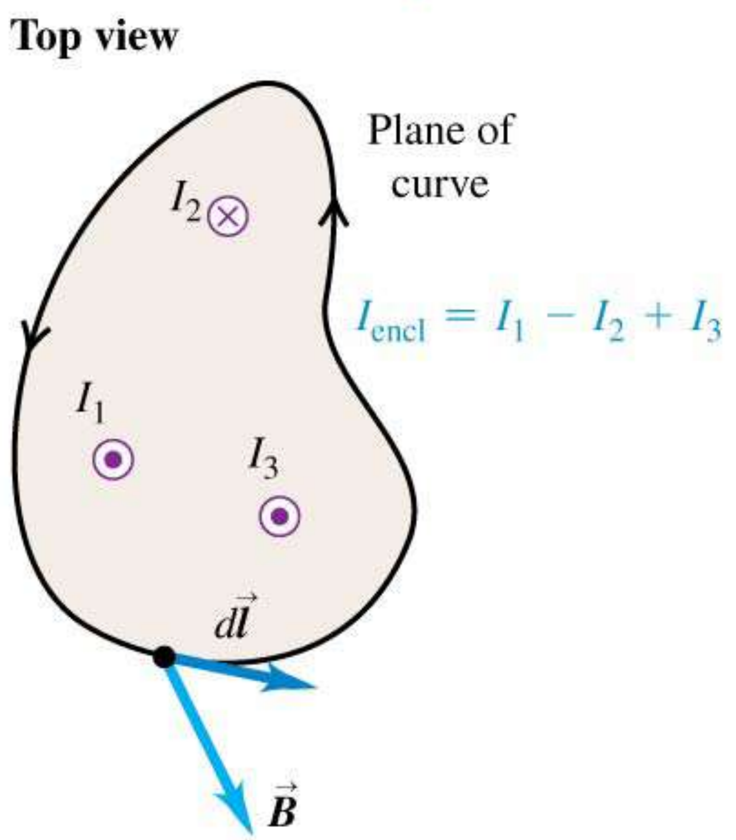
line integral around
a closed path

$$\oint \boldsymbol{B}.\,d\boldsymbol{l} = \mu_0 \boldsymbol{I}_{\mathbf{encl}}$$

net current
enclosed by path

Scalar product of magnetic field
and vector segment of path

'magnetic constant’
or ‘permeability of free space’



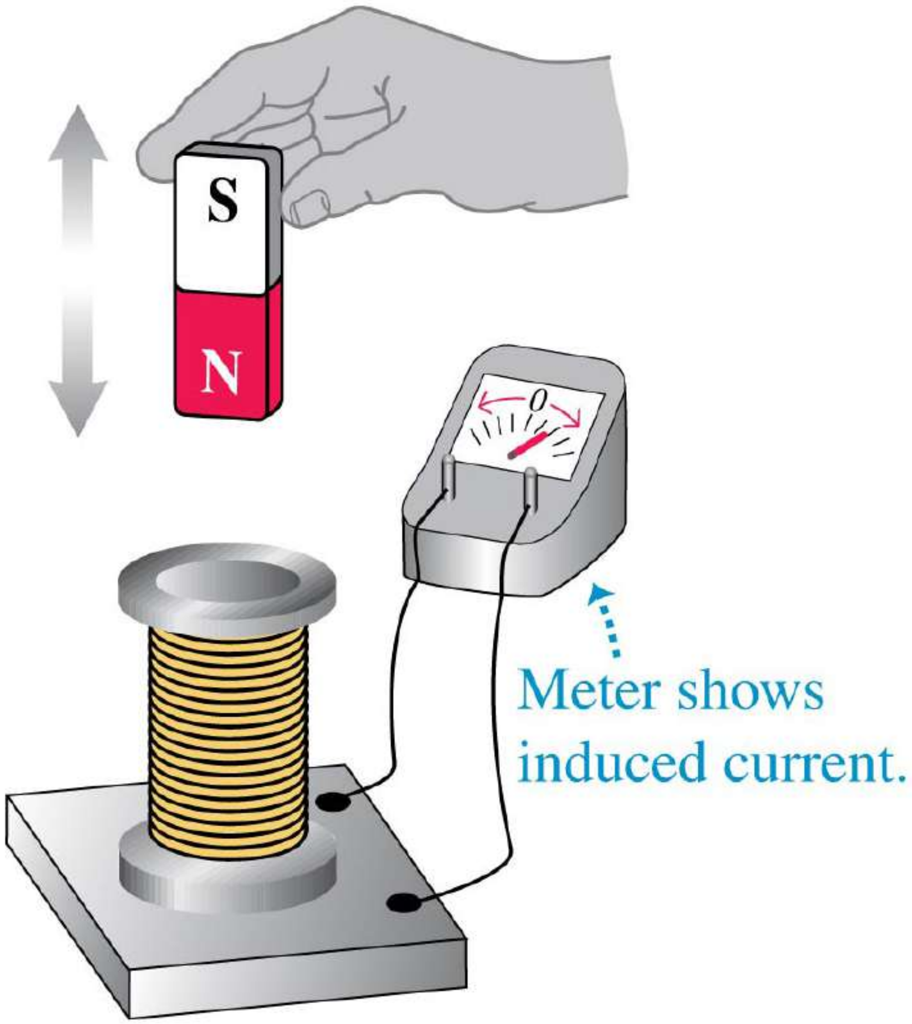
- I_{encl} is the algebraic sum of the currents enclosed or linked by the integration path, with the sum evaluated by using the right-hand sign rule.
- Reminder that μ_0 is permeability of free space, $4\pi \times 10^{-7}$ Hm^{-1}.

Induction experiment

When the magnetic field is constant and the shape, location, and orientation of the coil does not change, no current is induced in the coil.

When we move the magnet either toward or away from the coil the meter shows current in the circuit, but only while the magnet is moving.

This is known as an **induced current**, and the corresponding emf required to cause this current is called an **induced emf**.



Faraday's Law

When the magnetic flux through a single closed loop changes with time, there is an induced emf that can drive a current around the loop:

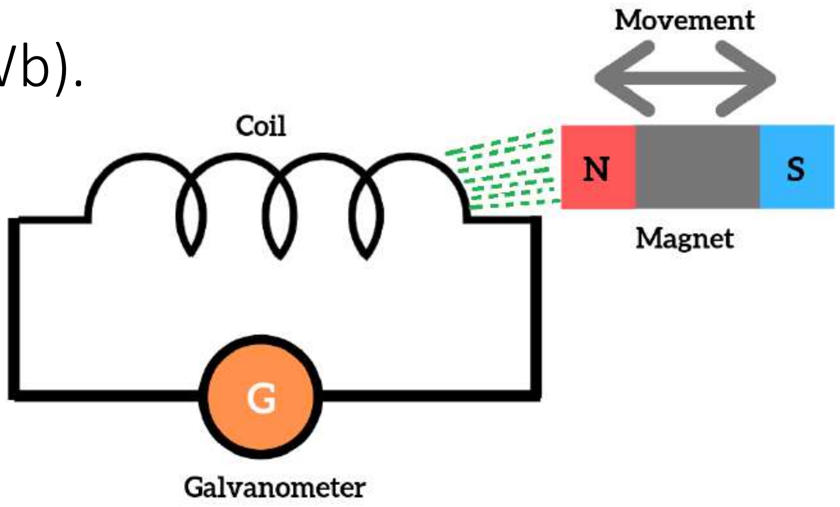
induced emf in a closed loop

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

negative of time rate of change of magnetic flux through the loop

Reminder that the unit of magnetic flux, Φ_B, is the weber (Wb).

1 V = 1 Wb s^{-1}, where 1 Wb = 1 T m^2



Induced electric fields

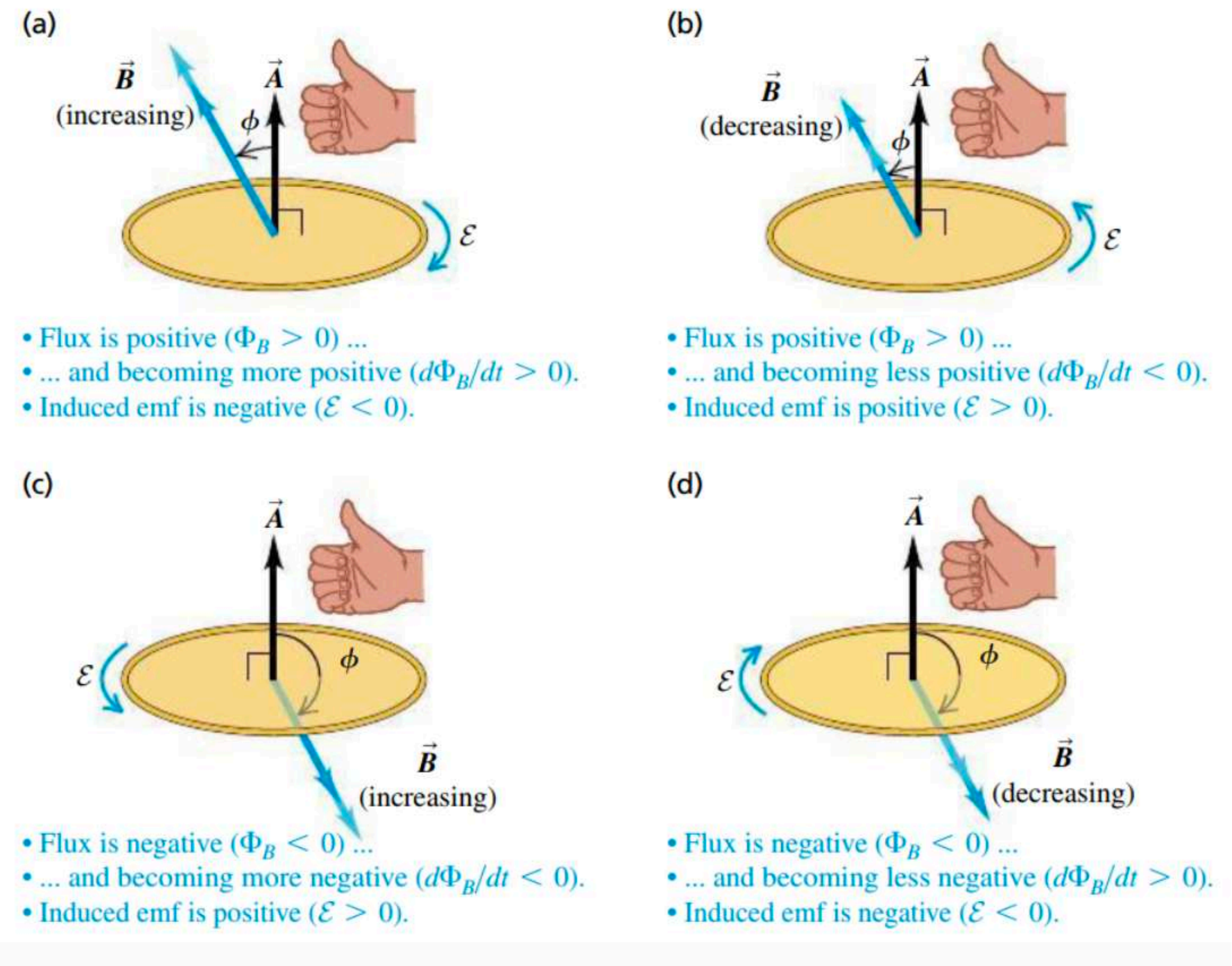
What about the force making the charges move through the wire? Its not a magnetic force since its not in a magnetic field – there is also an **induced electric field** caused by the changing magnetic flux through the stationary conductor.

Faraday's law for a stationary integration path:

$$\overbrace{\oint \vec{E} \cdot d\vec{l}}^{\text{Line integral of electric field around path}} = \underbrace{-\frac{d\Phi_B}{dt}}_{\text{Negative of the time rate of change of magnetic flux through path}}$$

Note: Our original formulation of Faraday's Law is for a stationary conductor. When a conductor is moving, motional emf can be defined as $\mathcal{E} = \oint (v \times B) \cdot dl$.

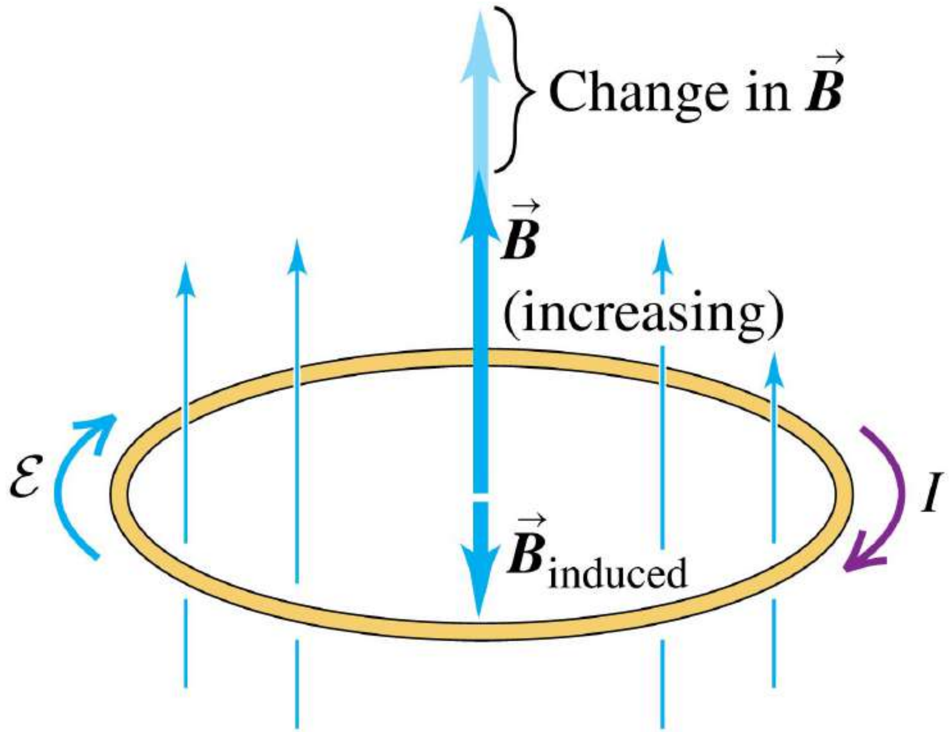
Faraday’s Law



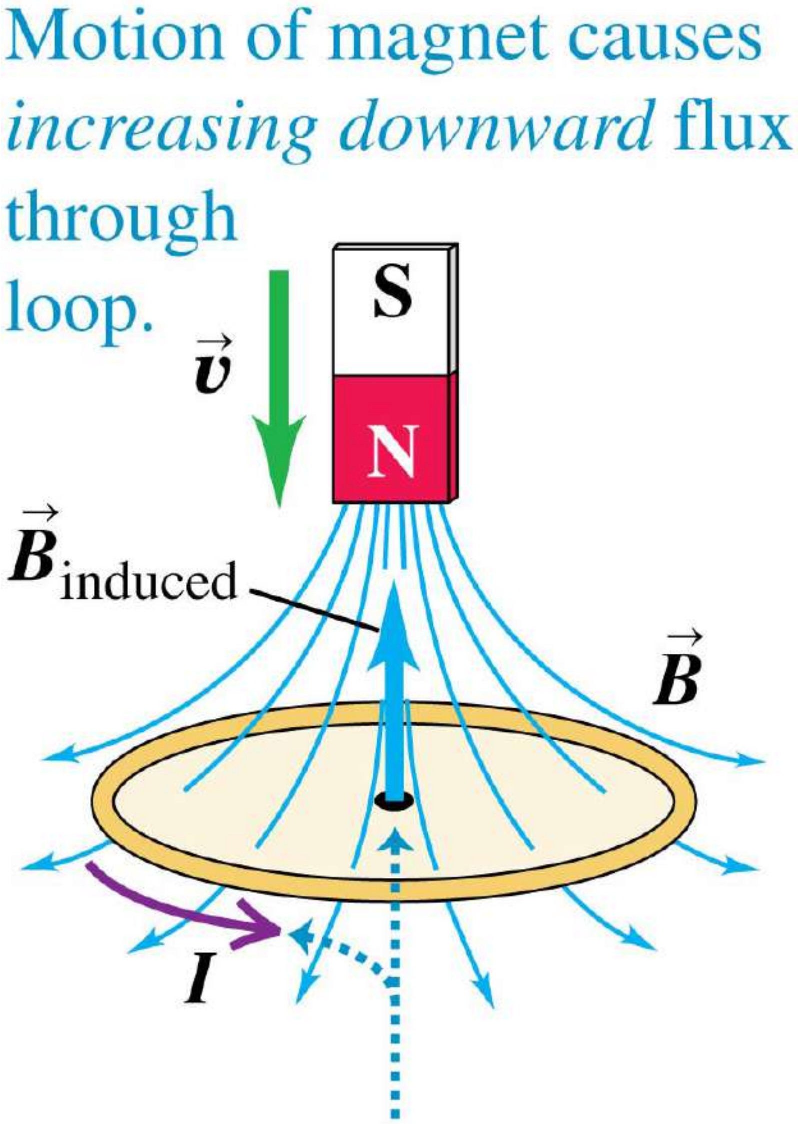
Lenz' Law

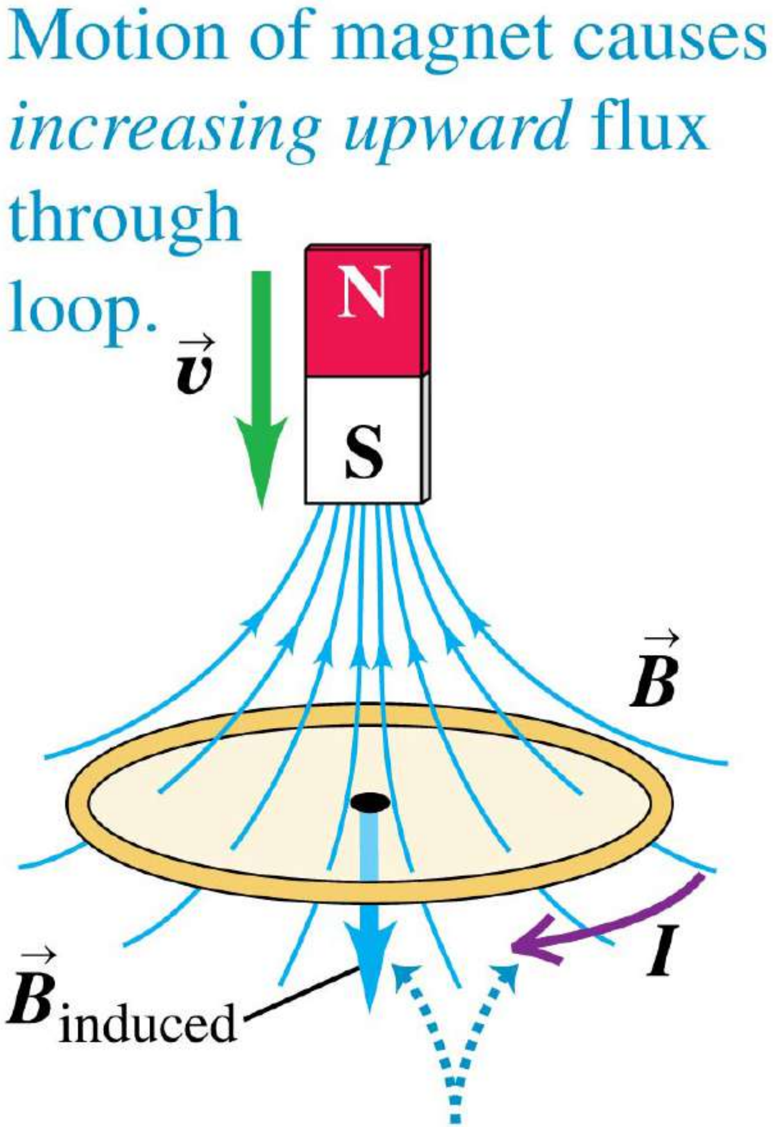
"The direction of any magnetic induction effect is such as to oppose the cause of the effect."

In the figure, there is a uniform magnetic field through the coil. The magnitude of the field is increasing, so there is an induced emf driving a current.



Lenz' Law


Motion of magnet causes *increasing downward* flux through loop.
S
N
$\vec{v}$
$\vec{B}_{\text{induced}}$
$\vec{B}$
I


Motion of magnet causes *increasing upward* flux through loop.
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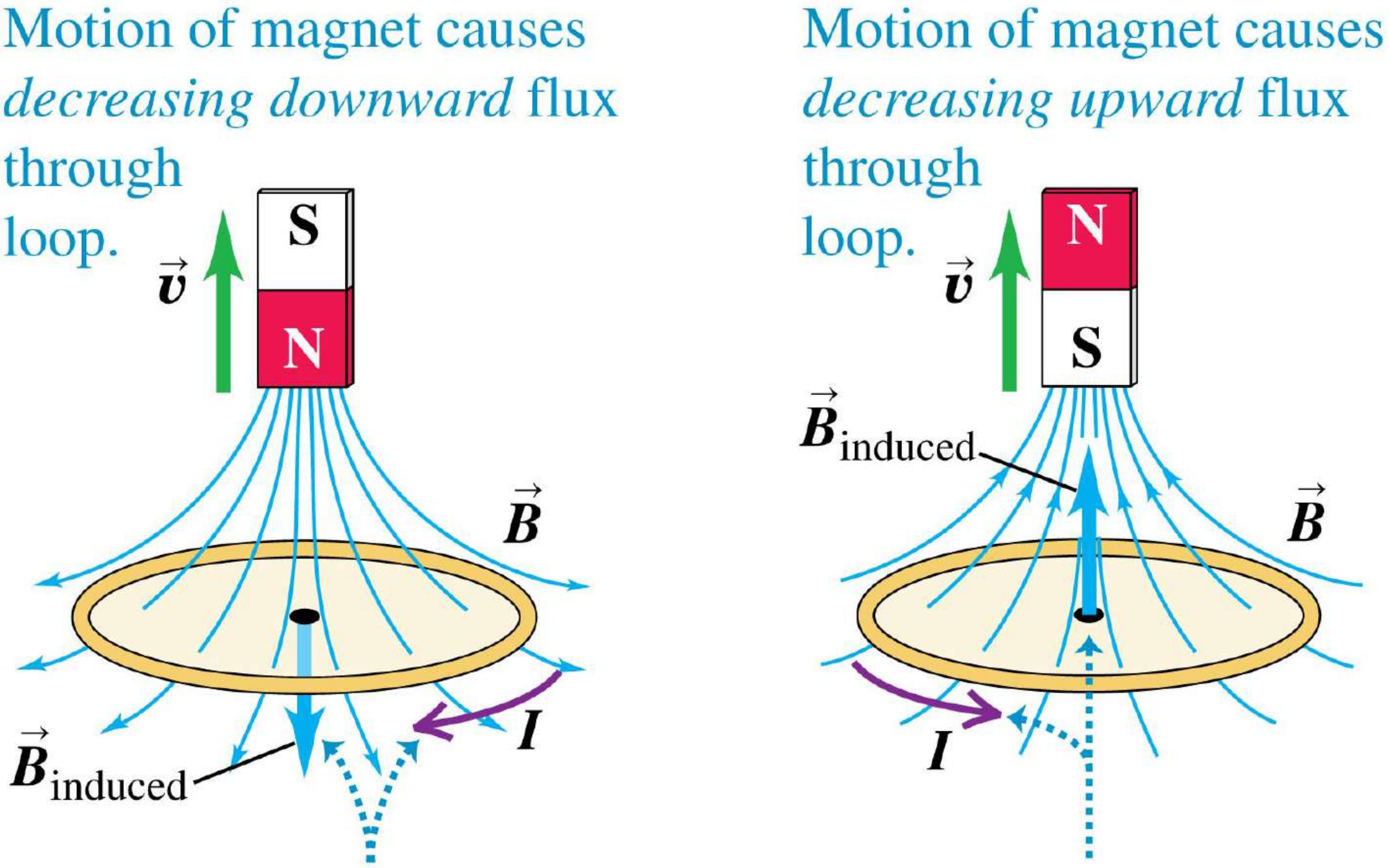
Lenz' Law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Motion of magnet causes *decreasing upward* flux through loop.
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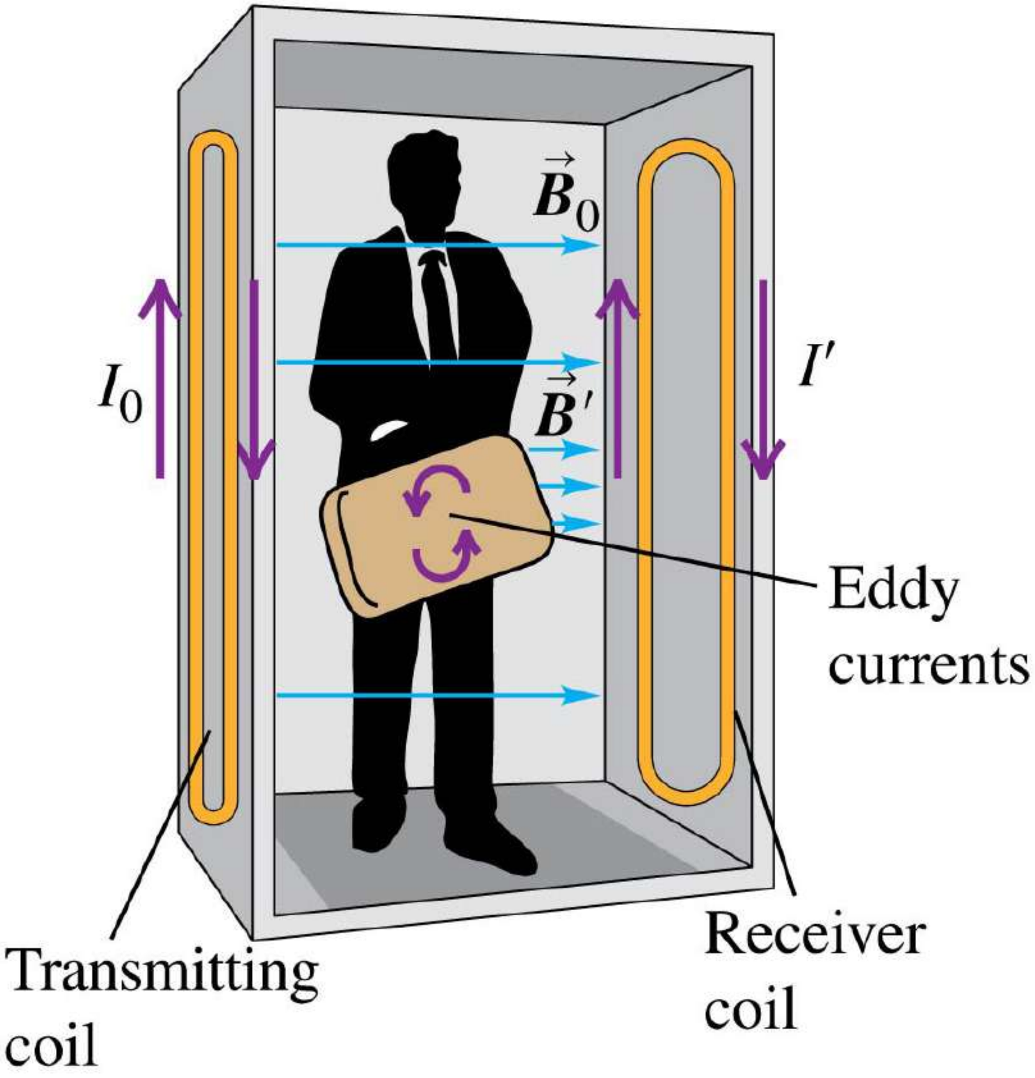
Real world: airport security

When a piece of metal moves through a magnetic field or is located in a changing magnetic field, **eddy currents** of electric current are induced.

The metal detectors used at airport security checkpoints operate by detecting eddy currents induced in metallic objects.

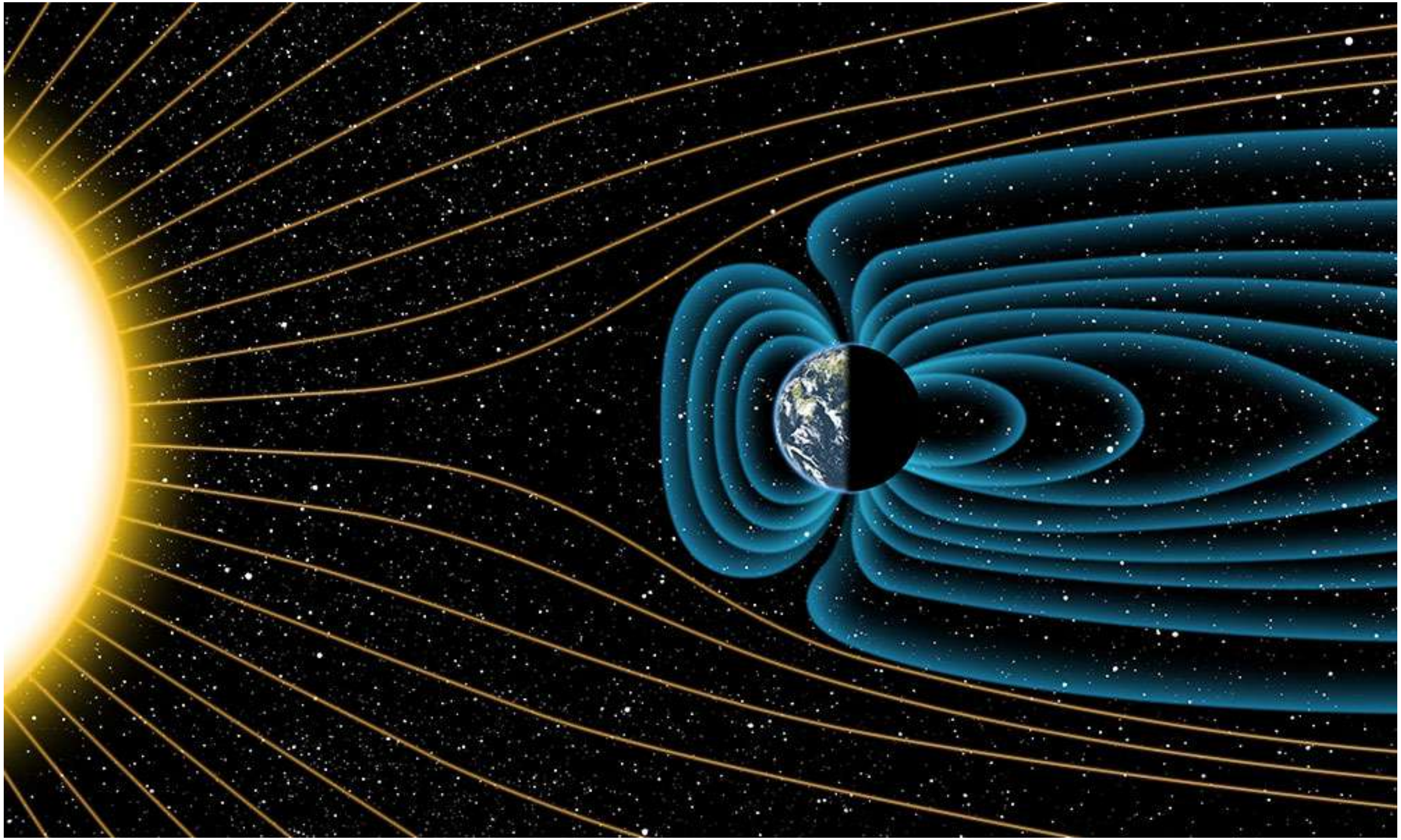


Real world: alternators

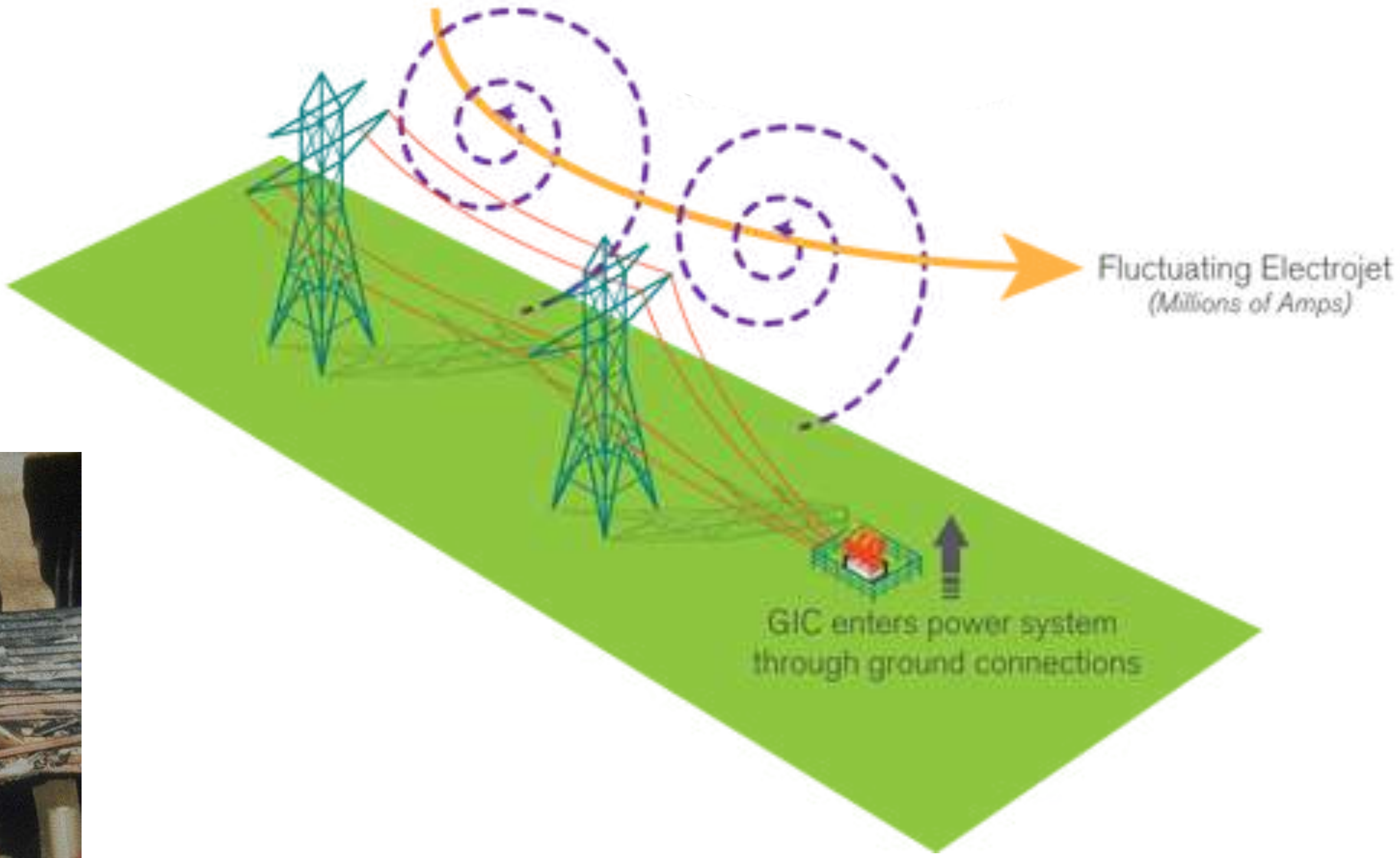
- A commercial alternator uses many loops of wire wound around a barrel-like structure called an armature.
- The resulting induced emf is far larger than would be possible with a single loop of wire.
- If a coil has N identical turns and if the flux varies at the same rate through each turn, total emf is:

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

Real world: geomagnetic storms



Real world: geomagnetic storms



Maxwell's equations of electromagnetism

All the relationships between electric and magnetic fields and their sources are summarised by four equations, called **Maxwell's equations**.

The first Maxwell equation is **Gauss's law for electric fields** (Lecture 3):

Gauss's law for $\vec{E}$:

$$\overbrace{\oint \vec{E} \cdot d\vec{A}}^{\text{Flux of electric field through a closed surface}} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

Q_{encl}: Charge enclosed by surface

ϵ_0: Electric constant

Maxwell's equations

All the relationships between electric and magnetic fields and their sources are summarized by four equations, called **Maxwell's equations**.

The second Maxwell equation is **Gauss's law for magnetic fields** (Lecture 5):

Gauss's law for $\vec{B}$:

Flux of magnetic field through any closed surface ...

$$\underbrace{\oint \vec{B} \cdot d\vec{A}} = 0$$

... equals zero.

Maxwell’s equations

All the relationships between electric and magnetic fields and their sources are summarized by four equations, called **Maxwell’s equations**.

The third Maxwell equation is **Faraday’s law of induction**:

Faraday’s law for a stationary integration path:

$$\overbrace{\oint \vec{E} \cdot d\vec{l}}^{\text{Line integral of electric field around path}} = \underbrace{-\frac{d\Phi_B}{dt}}_{\text{Negative of the time rate of change of magnetic flux through path}}$$

Maxwell's equations

All the relationships between electric and magnetic fields and their sources are summarized by four equations, called **Maxwell's equations**.

The fourth Maxwell equation is **Ampere's law**, including displacement current:

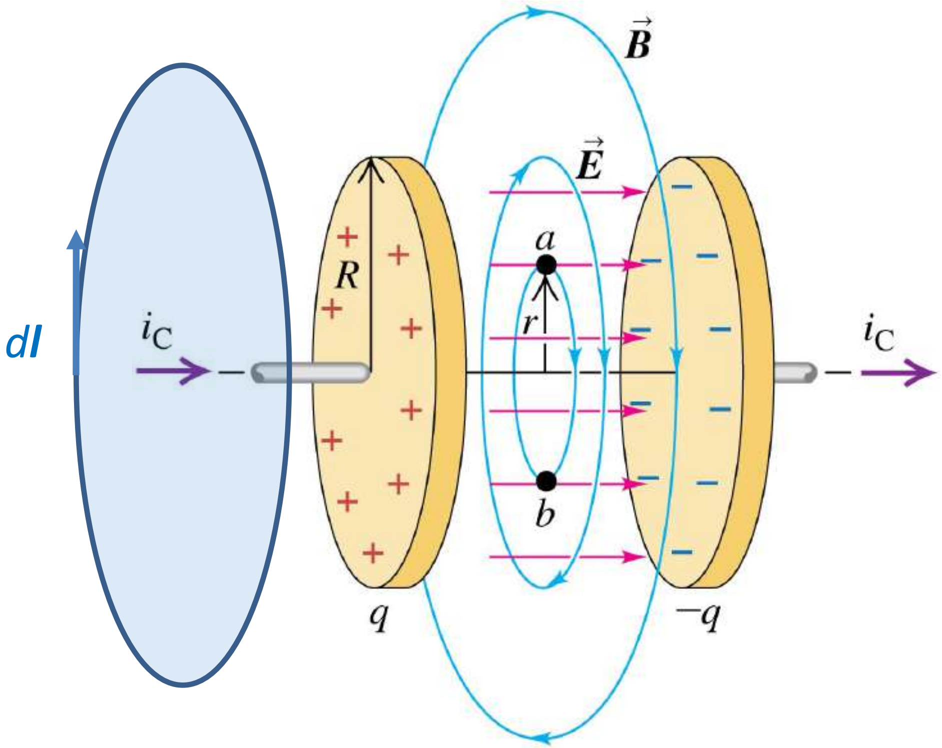
Ampere's law for a stationary integration path:

$$\overbrace{\oint \vec{B}\cdot d\vec{l}}^{\text{Line integral of magnetic field around path}} = \underbrace{\mu_0}_{\text{Magnetic constant}}\left(\underbrace{i_C}_{\text{Conduction current through path}} + \underbrace{\overbrace{\epsilon_0}^{\text{Electric constant}}\overbrace{\frac{d\Phi_E}{dt}}^{\text{Time rate of change of electric flux through path}}}_{\text{Displacement current through path}}\right)_{\text{encl}}$$

Displacement current

Ampere’s law that we showed earlier is *incomplete*, as can be shown by considering the process of charging a capacitor.

When a capacitor is charging, the electric field is increasing between the plates. We can define a fictitious displacement current i_D in the region between the plates:



Displacement current through an area

$$i_D = \epsilon \frac{d\Phi_E}{dt}$$

Time rate of change of electric flux through area

Permittivity of material in area

Maxwell's equations

$$\boxed{c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}}$$

$$\oint E.dA = \frac{q}{\varepsilon_0}$$

$$\oint B.dA = 0$$

$$\oint E.dl = -\frac{d\Phi_B}{dt}$$

$$\oint B.dl = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell's equations

$$\boxed{c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}}$$

$$\oint E.dA = \frac{q}{\varepsilon_0}$$

$$\nabla.E = \frac{\rho}{\varepsilon_0}$$

$$\oint B.dA = 0$$

$$\nabla.B = 0$$

$$\oint E.dl = -\frac{d\Phi_B}{dt}$$

$$\nabla\times E = -\frac{\partial B}{\partial t}$$

$$\oint B.dl = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\nabla\times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

Maxwell's equations

There is a remarkable symmetry in Maxwell's equations.

In empty space where there is no charge, the first two equations are identical in form.

The third equation says that a changing magnetic flux creates an electric field, and the fourth says that a changing electric flux creates a magnetic field.

In empty space there are no charges, so the fluxes of $\vec{E}$ and $\vec{B}$ through any closed surface are equal to zero.

$$\oint \vec{E} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

In empty space there are no conduction currents, so the line integrals of $\vec{E}$ and $\vec{B}$ around any closed path are related to the rate of change of flux of the other field.

Maxwell’s equations

Originally Answered: What are the real life applications of Maxwell eqations?

If you use electricity, then you are using technology that was designed using Maxwell’s equations. Similarly, if you use a cellphone, then you are using designs based upon Maxwell’s equations. Maxwell’s equations govern the design of alternators and generators that you may use to keep your lights on, and computer manufacturers who arrange components in a computer need to understand how Maxwell’s equations describe the way that the electronic components will interfere with one another.

https://www.wired.com/story/get-to-know-maxwells-equationsyoure-using-them-right-now/

Summary

- Ampere’s Law CH 28
- Faraday’s Law and Lenz’ Law CH 29
- Maxwell’s Equations CH 29

Next time: Inductance and more circuits CH 30/31