PY1E04

Introduction to Physics (Electromagnetism)

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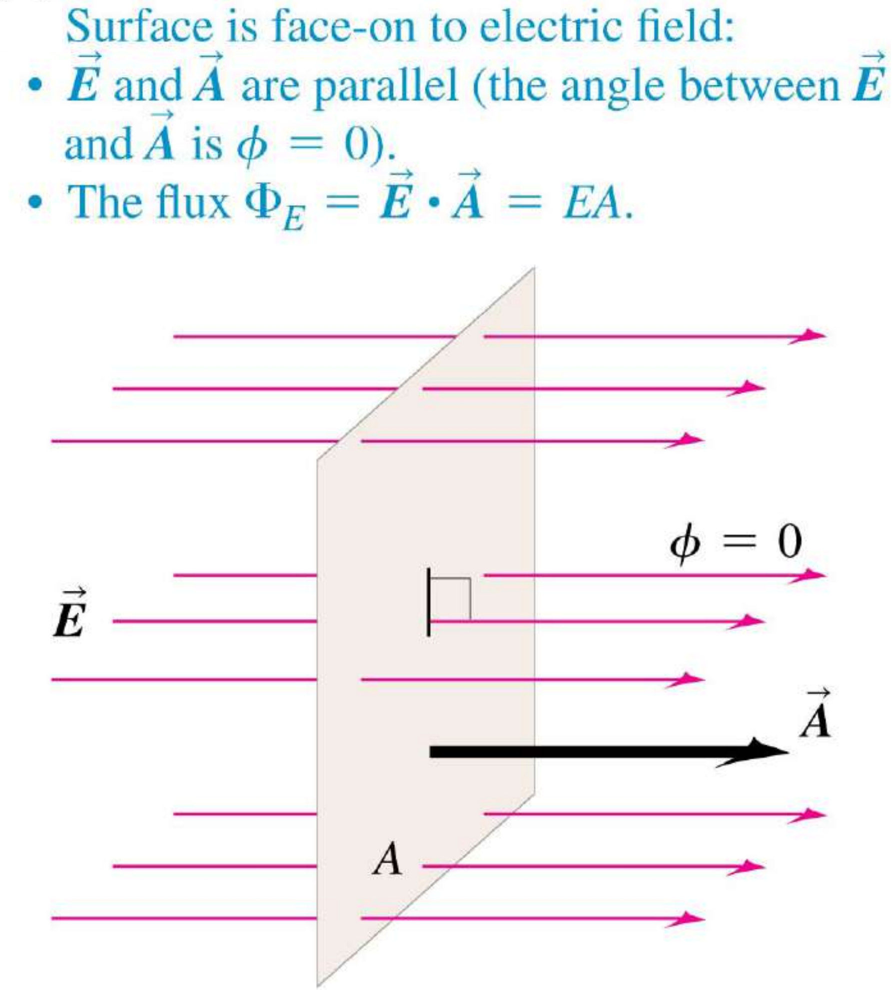
Reminder

Electric flux is the rate of flow of the **electric field** through a given **area**.

Consider a flat area perpendicular to a uniform electric field.
Here, $\Phi_E = E\,A$

- Increasing the area means that more electric field lines pass through the area, increasing the flux.
- A stronger field means more closely spaced lines, and therefore more flux.

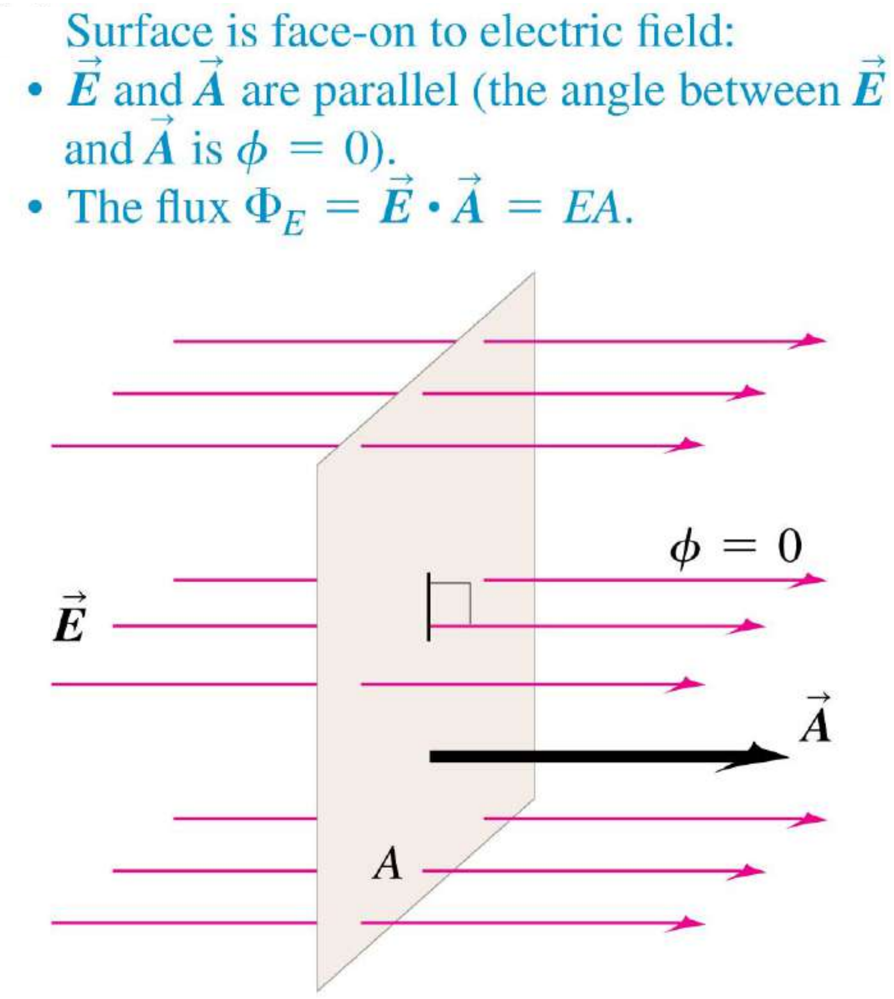


Calculating electric flux

Electric flux is the rate of flow of the **electric field** through a given **area**.

Consider a flat area perpendicular to a uniform electric field.
Here, $\Phi_E = E\ A$

The **SI unit** of Φ_E is 1 N m^2 C^{-1}
(since unit of E is N/C and of A is m^2)

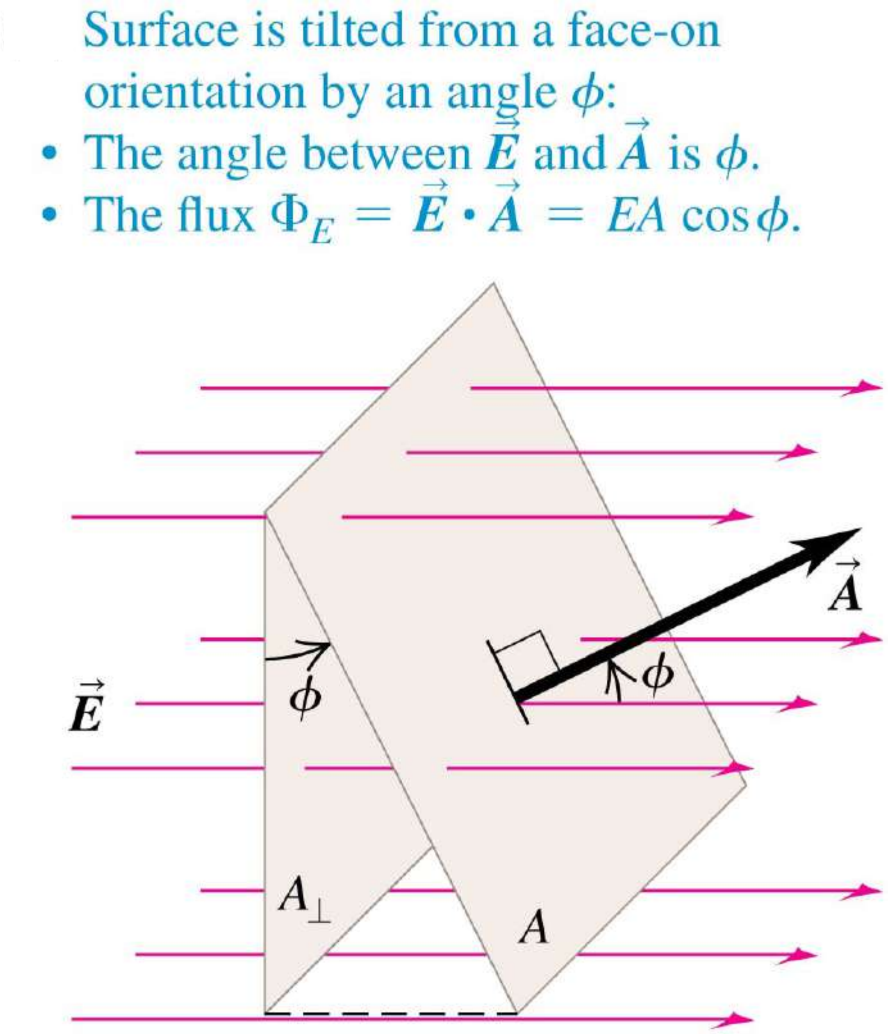


Calculating electric flux

If the area is not perpendicular to the field, then fewer field lines pass through it.

Here,

$$\Phi_E = E\ A \cos \emptyset$$



Calculating electric flux

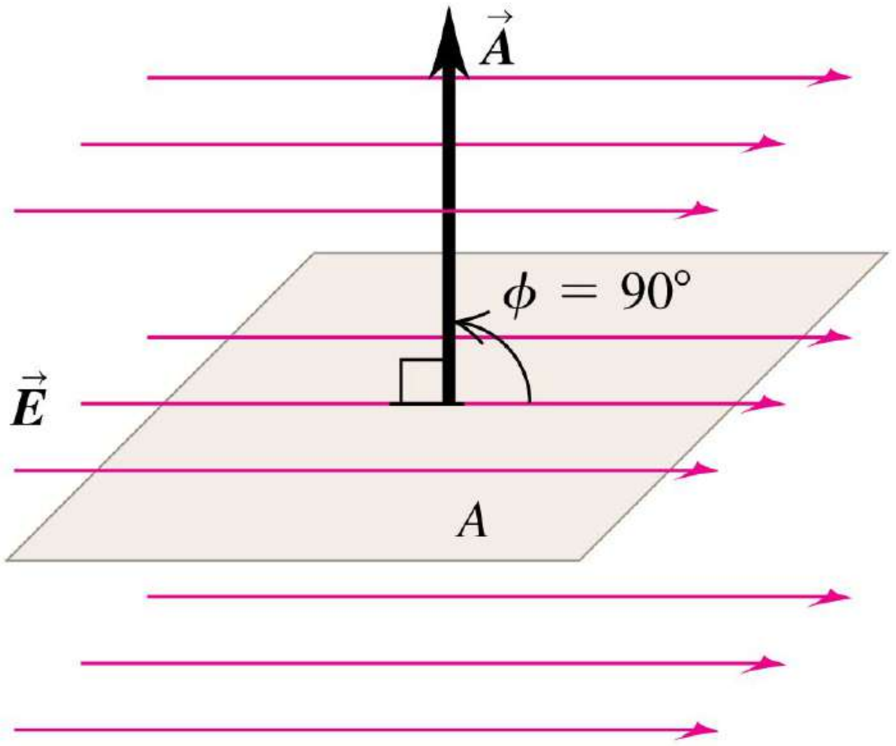
If the area is edge-on to the field, then the area is perpendicular to the field and the flux is zero.

Here,

$$\Phi_E = E\,A\cos 90° = 0$$

(c) Surface is edge-on to electric field:

- $\vec{E}$ and $\vec{A}$ are perpendicular (the angle between $\vec{E}$ and $\vec{A}$ is $\phi = 90°$).
- The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos 90° = 0$.



Calculating electric flux

In general, the flux through a surface must be computed using a **surface integral** over the area:

$$\Phi_E = \int E\cos\phi \, dA = \int E_\perp \, dA = \int \vec{E} \cdot d\vec{A}$$

Electric flux through a surface

Magnitude of electric field $\vec{E}$

Component of $\vec{E}$ perpendicular to surface

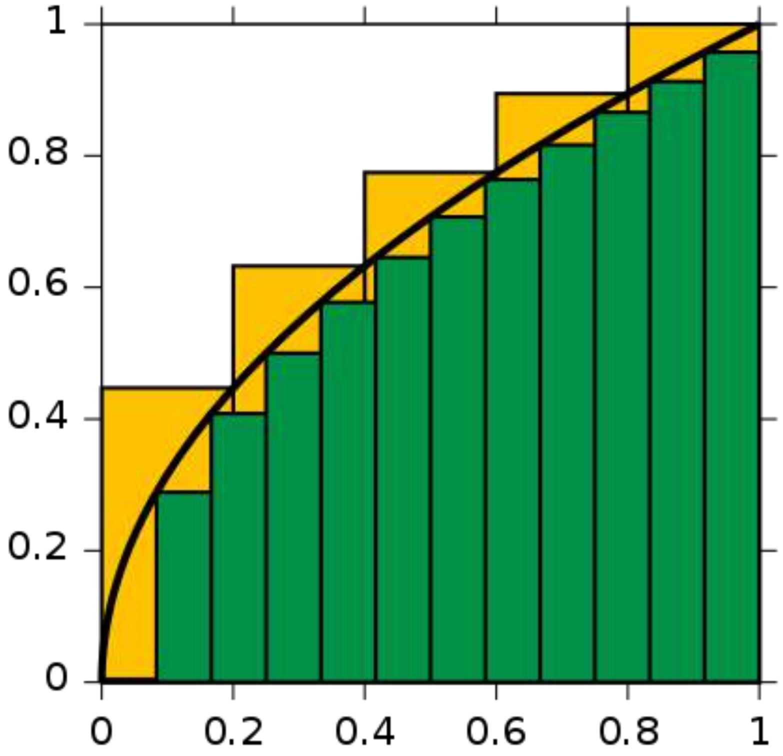
Angle between $\vec{E}$ and normal to surface

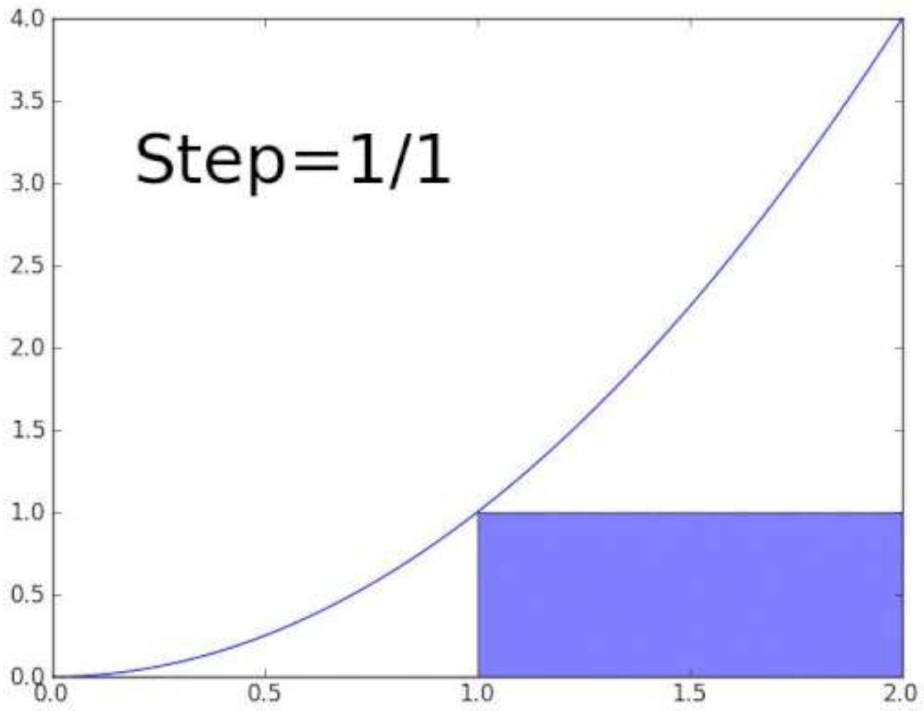
Element of surface area

Vector element of surface area

Reminder: integrals

Way of describing displacement, area, volume, etc by combining infinitesimal data.





https://en.wikipedia.org/wiki/Integral

Gauss’s law

Let Q_{encl} be the total charge enclosed by a surface.

Gauss’s law states that the total electric flux through a closed surface is proportional to the total (net) electric charge inside the surface

Gauss’s law:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

Total charge enclosed by surface

Electric flux through a closed surface of area A = surface integral of $\vec{E}$

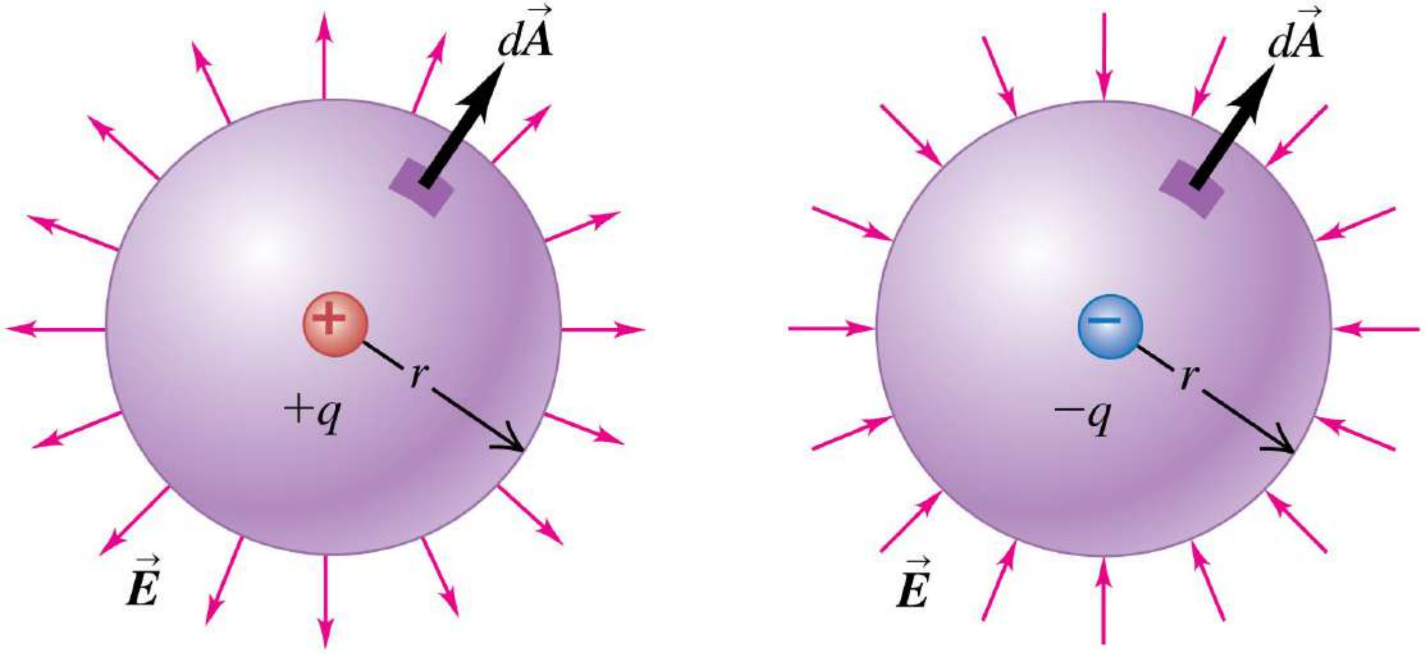
Electric constant

Gauss's Law

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

Completely equivalent to Coulomb's law, it provides a different way to express the relationship between electric charge and electric field.

… a relationship between the field at all the points on the surface and the total charge enclosed within the surface.



$$\Phi_E = EA = \frac{q}{\varepsilon_0}$$

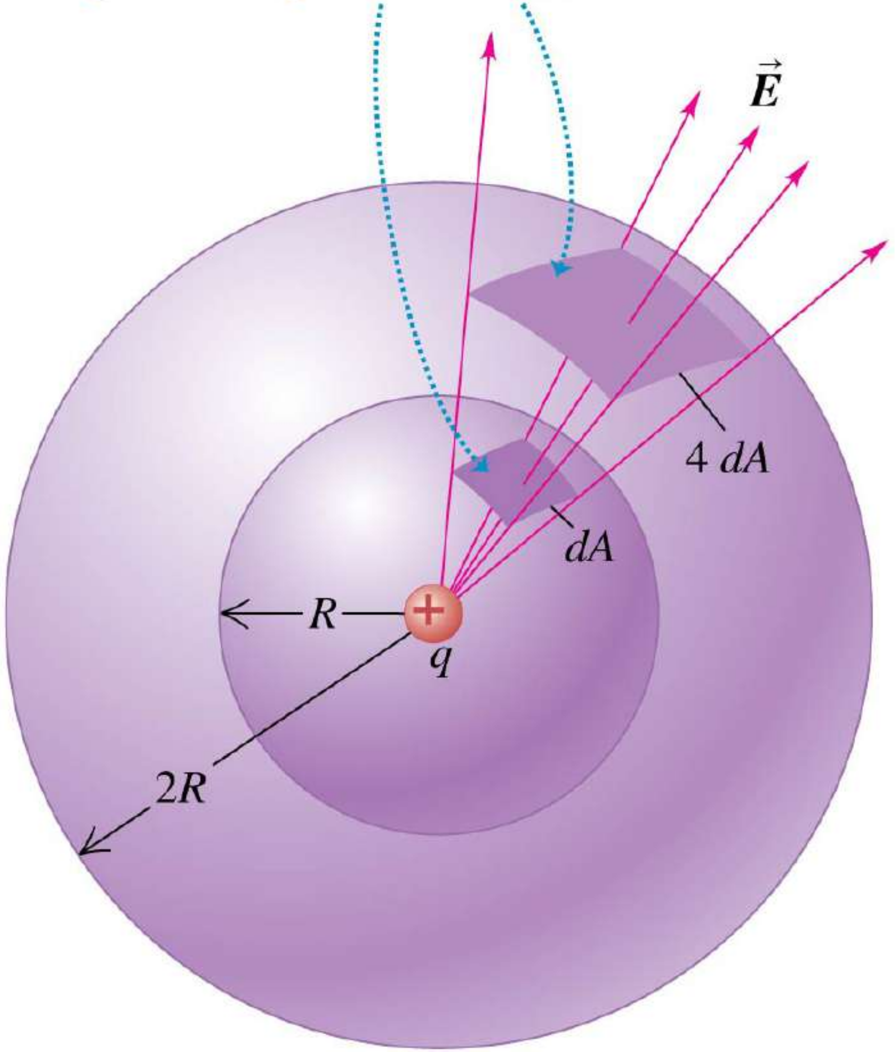
$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

Gauss's law

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

The same number of field lines and the same flux pass through both of these area elements.



Area dA of a sphere of radius R projected onto sphere of radius 2R.

Area of larger sphere is 4 dA, but the electric field magnitude is ¼ as great on the larger sphere as on the smaller sphere.

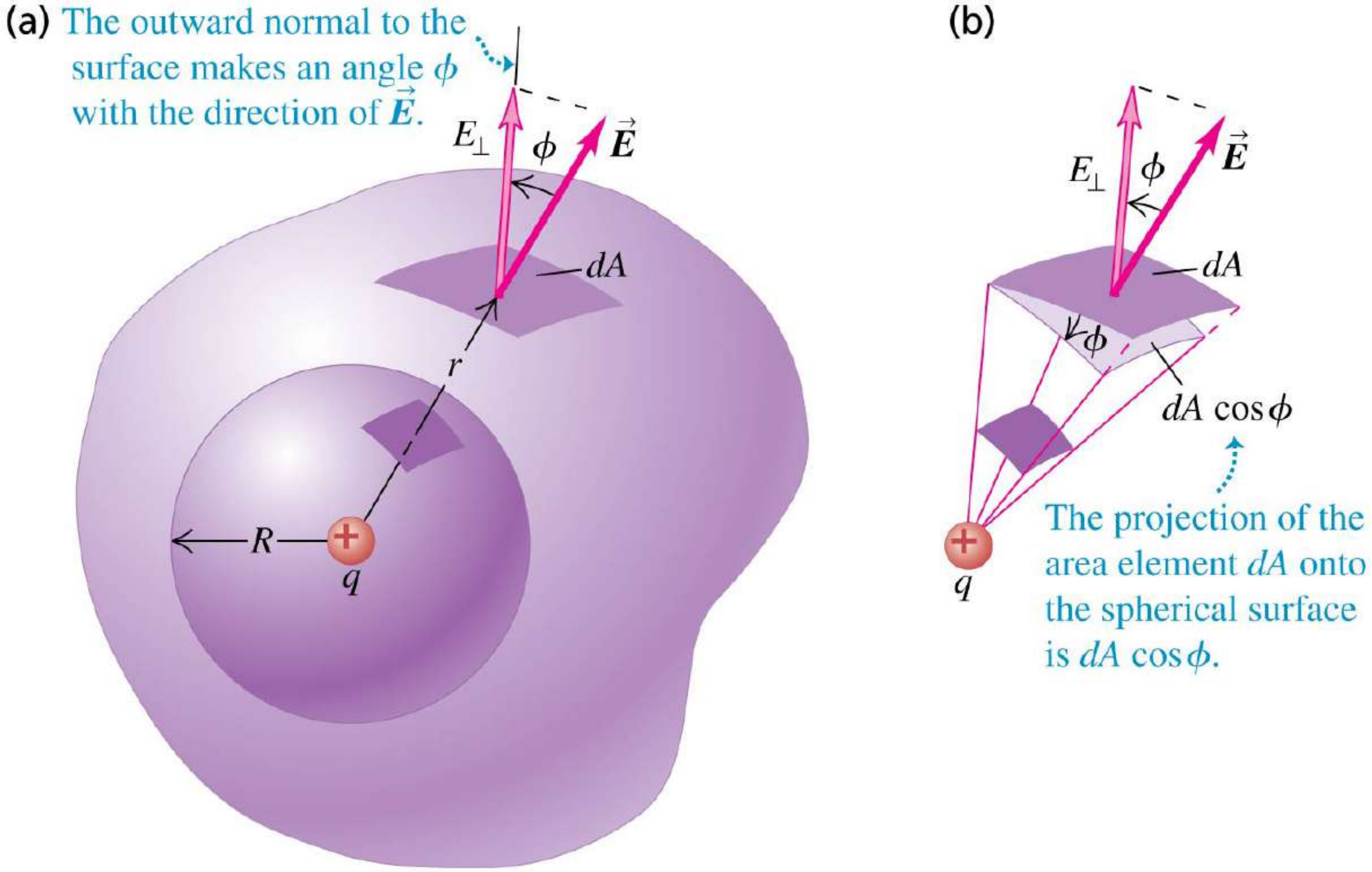
-> The electric flux is the same for both areas and is independent of the radius of the sphere.

$$\Phi_E = EA = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}(4\pi R^2) = \frac{q}{\epsilon_0}$$

Gauss's law

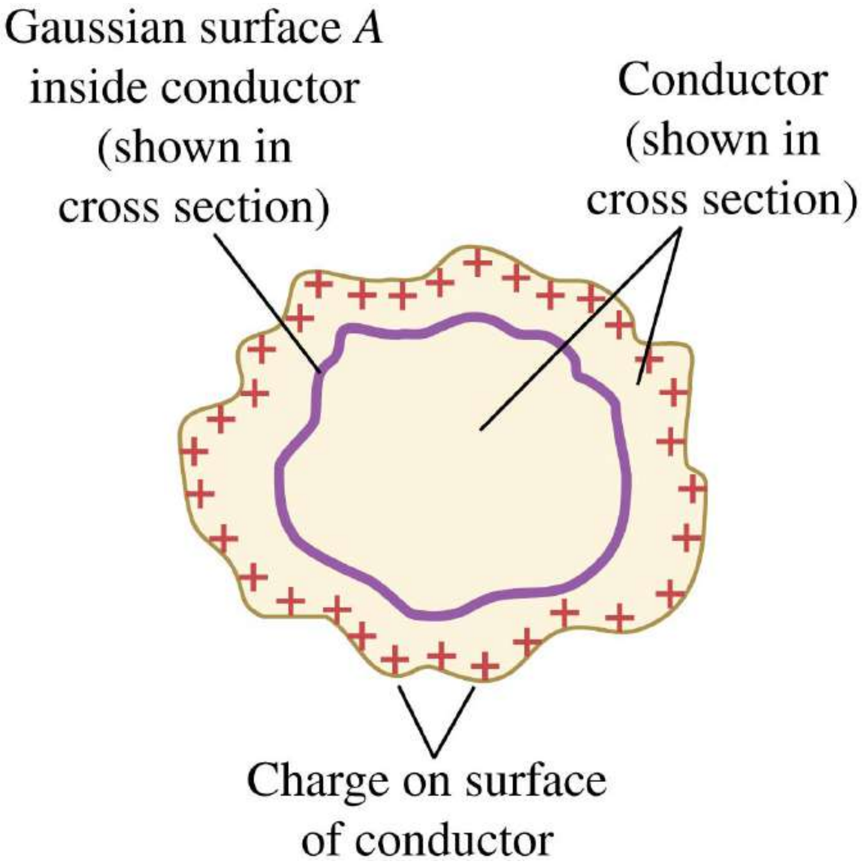
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

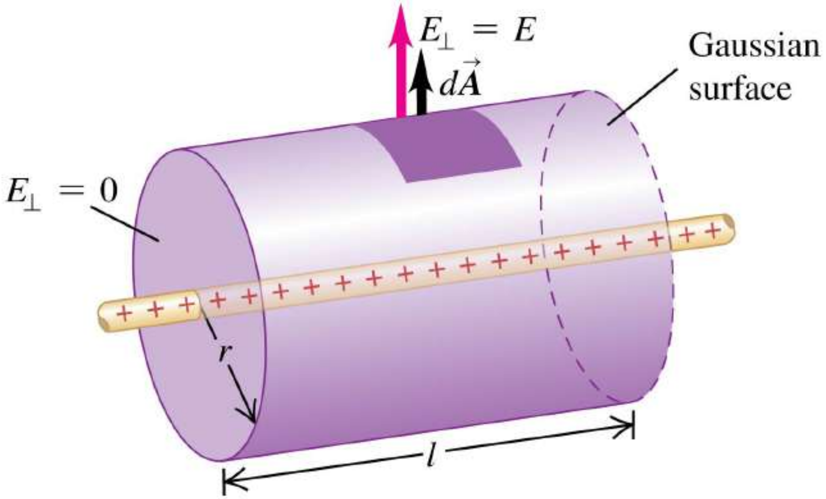
This holds for non-spherical surfaces too – can be a surface of any shape or size, provided only that it is a closed surface enclosing the charge q.

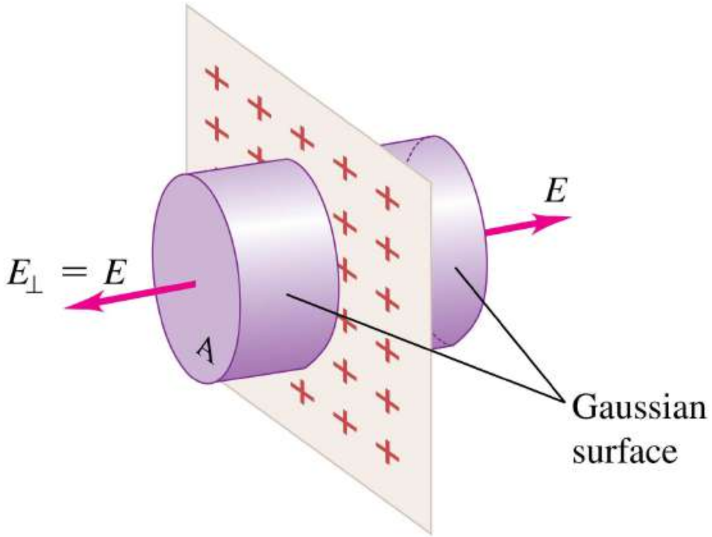


Applications of Gauss's Law

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$







$$E = 0$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

Gauss’s law in action

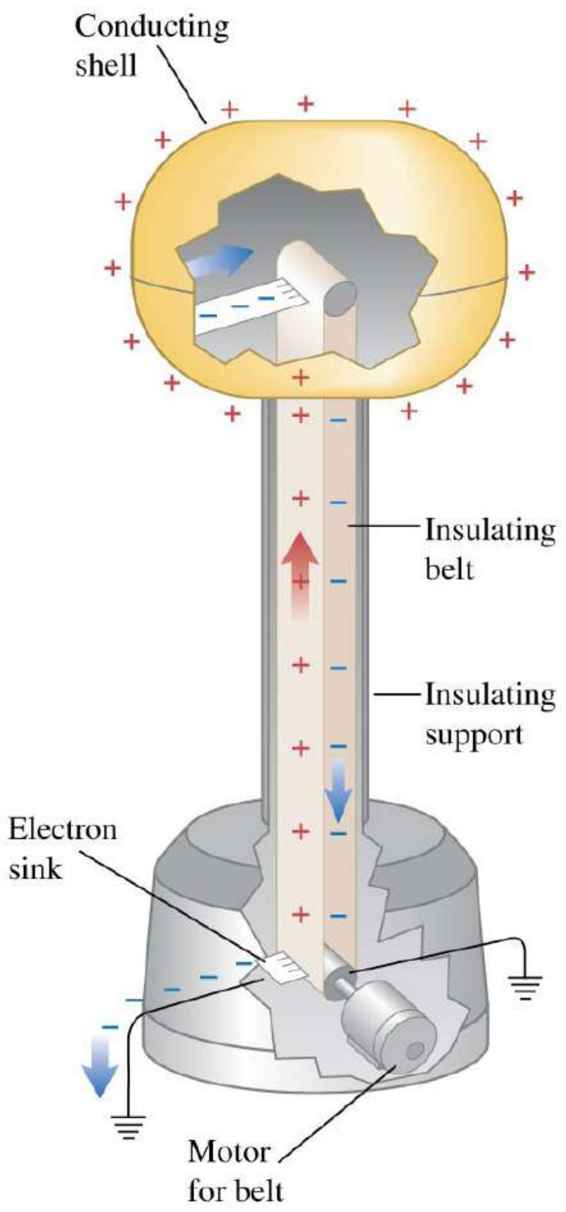
Van de Graaff generator

Gauss's law in action

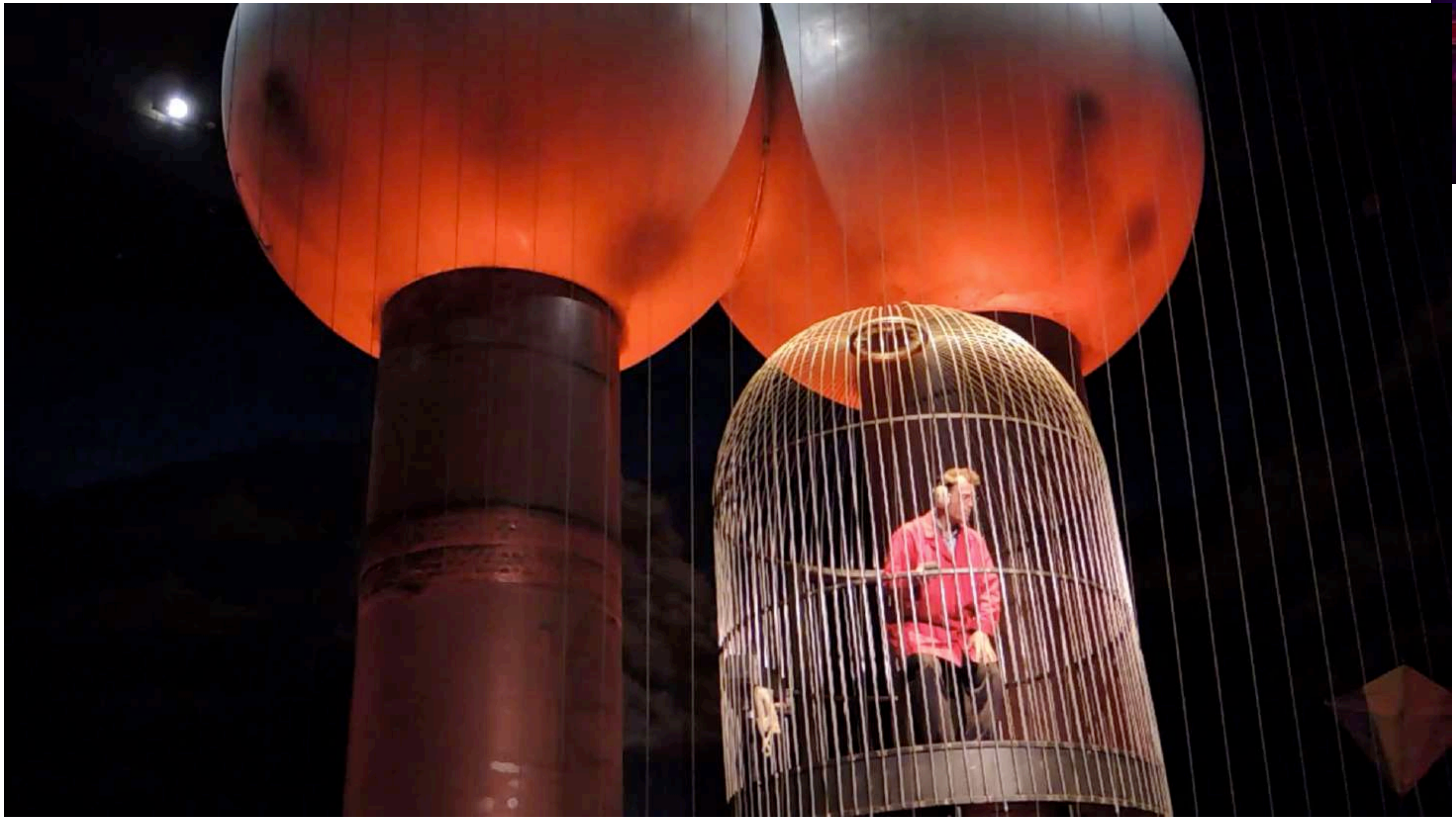
Van de Graaff generator

The electron sink at the bottom draws electrons from the belt, giving it a positive charge.

At the top the belt attracts electrons away from the conducting shell, giving the shell a positive charge.



Gauss’s law in action



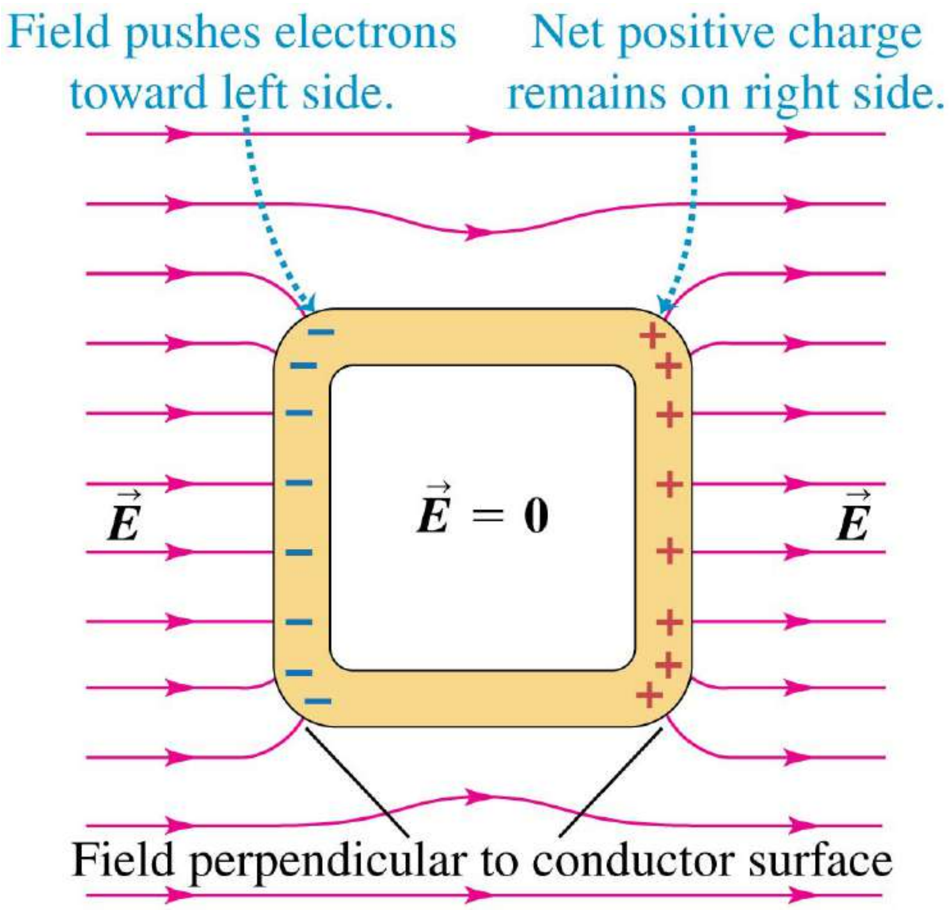
https://www.youtube.com/watch?v=vheyKyV8vSA

Gauss’s law in action

Faraday cage

A conducting box is immersed in a uniform electric field.

The field of the induced charges on the box combines with the uniform field to give zero total field inside the box.

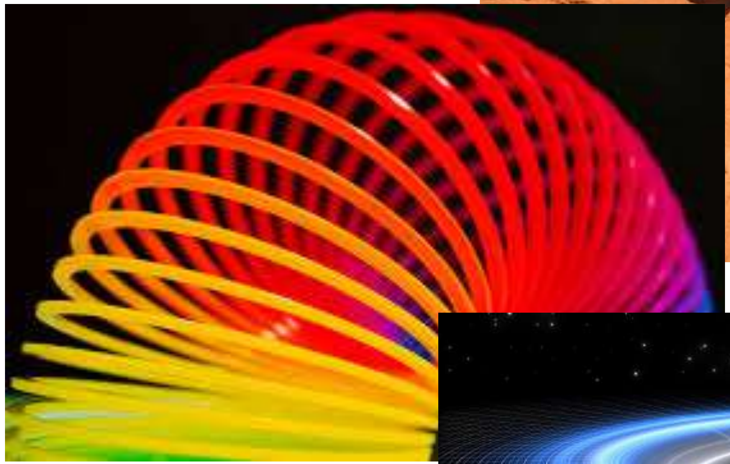


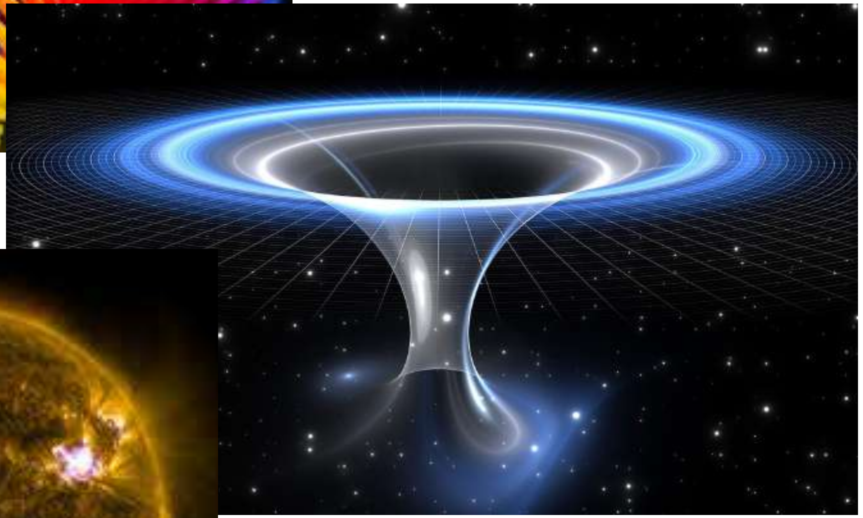
Potential energy

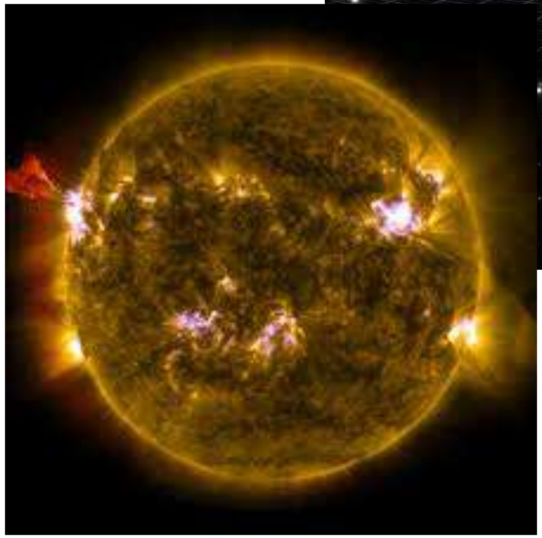
Potential energy is
the energy stored within an object,
due to the object's position,
arrangement or state.

You'll find it in e.g.,

- mechanics ($\frac{1}{2} * k * x^2$),
- gravitation ($m * g * h$),
- and electromagnetism.



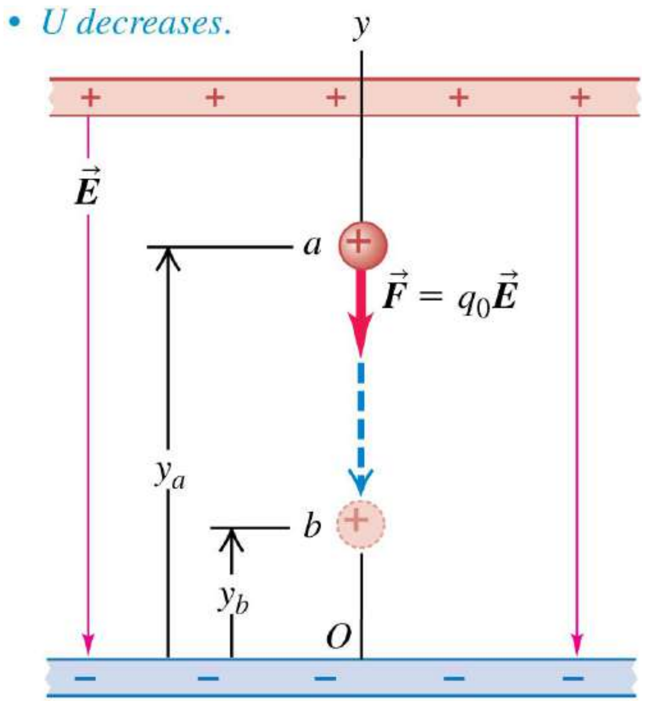




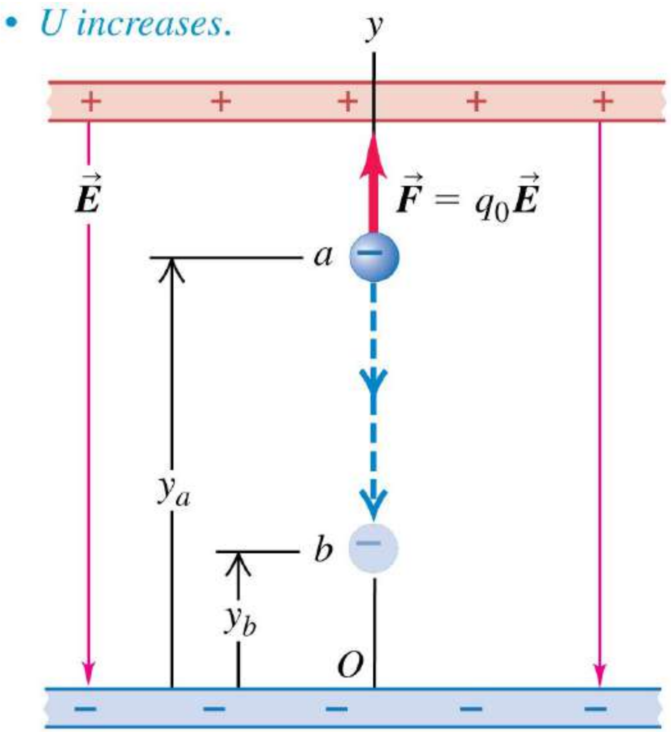
Electric potential energy

The downwards electric field between two charged parallel metal plates exerts a force.

For a positive charge moving downwards (attracting), the field does positive work on the charge
-> potential energy decreases



For a negative charge moving Downwards (repelling), the field does negative work on the charge
-> potential energy increases



Electric potential energy

Unit: Joule (J)

The electric potential energy of two point charges only depends on the distance between the charges.

$$\underbrace{U}_{\text{Electric potential energy of two point charges}} = \frac{1}{\underbrace{4\pi\epsilon_0}_{\text{Electric constant}}} \frac{\overbrace{q q_0}^{\text{Values of two charges}}}{\underbrace{r}_{\text{Distance between two charges}}}$$

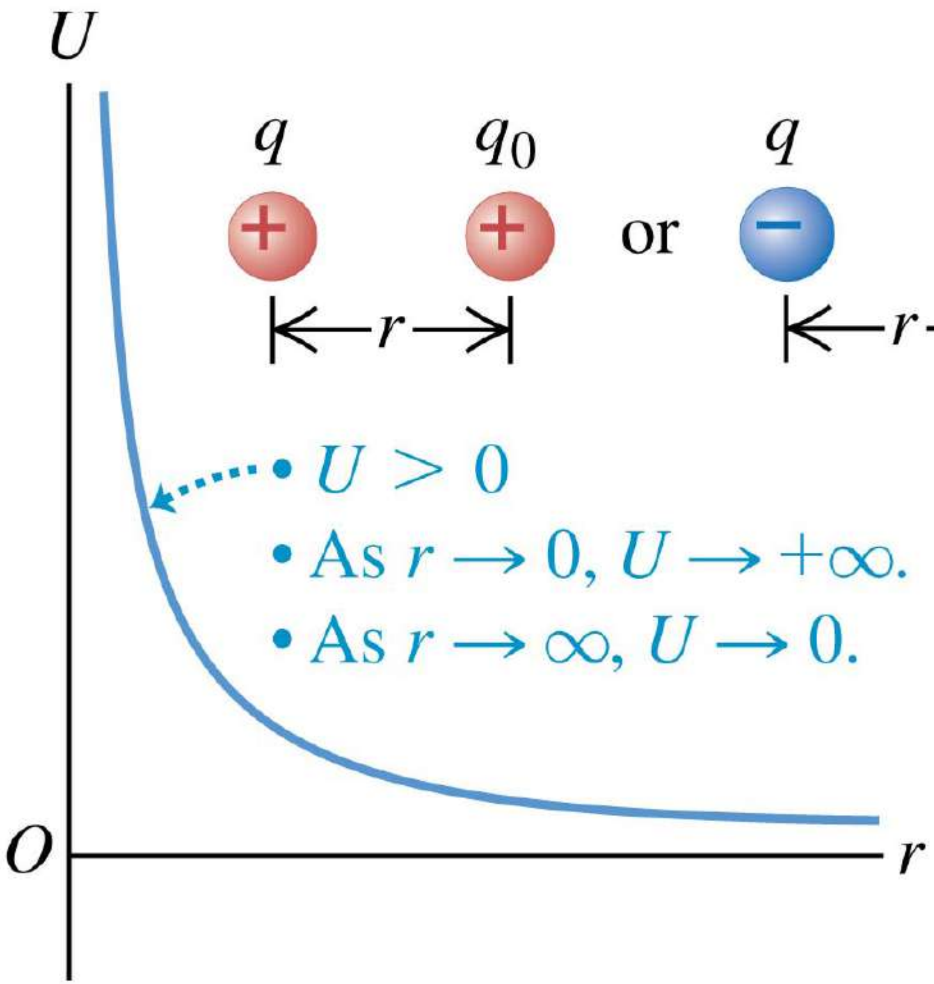
This equation is valid no matter what the signs of the charges are. U is defined to be zero when the charges are infinitely far apart.

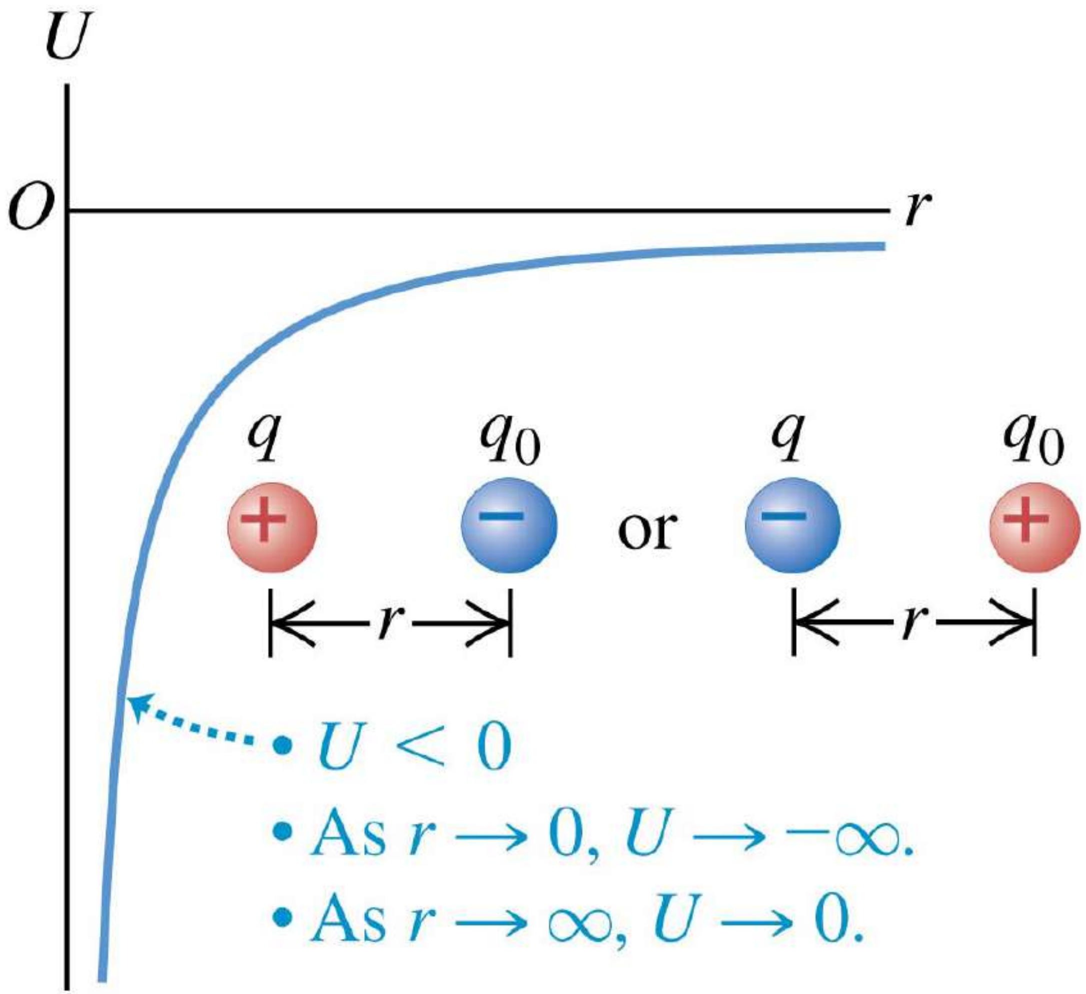
The force on an object is the negative of the derivative of U:

$F(x) = -\frac{dU}{dx}$ i.e., $U = -\int^{r} F.dr$

Electric potential energy

$$U = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r}$$





Electric potential energy

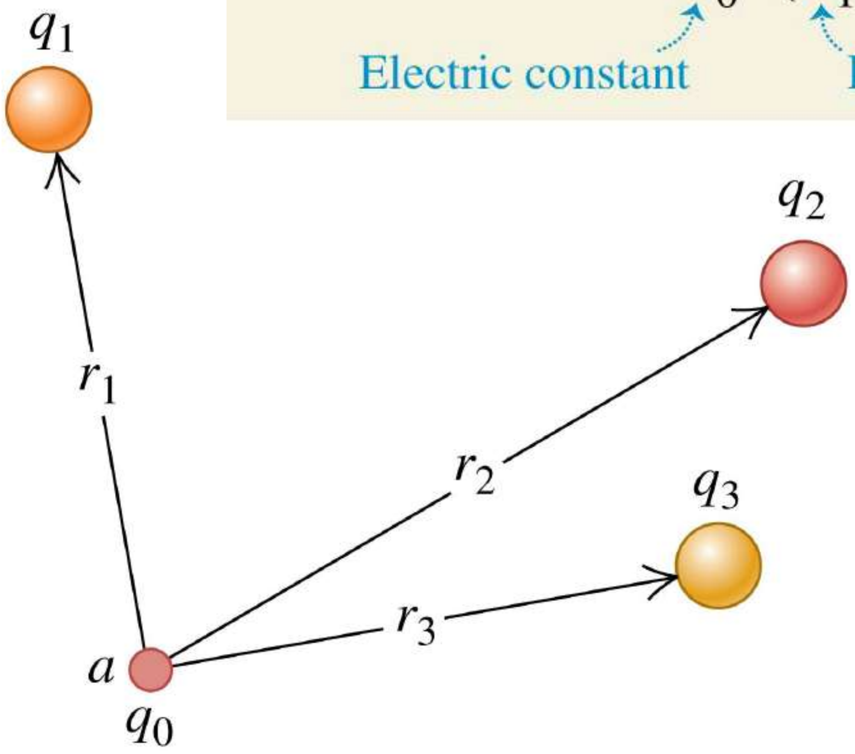
For several point charges, the electric potential energy is the algebraic sum:

Electric potential energy of point charge q_0 and collection of charges $q_1, q_2, q_3, \ldots$

$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

Electric constant

Distances from q_0 to $q_1, q_2, q_3, \ldots$



Electric potential

Electric potential is defined as **potential energy per unit charge:** $V = \frac{U}{q_0}$

For a single point charge:

Electric potential due to a point charge

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

Value of point charge

Distance from point charge to where potential is measured

Electric constant

Unit: Volts (V) = J C^{-1} = kg m^2 A^{-1} s^{-3}

Electric potential

For a single point charge:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

For a collection of charges:

$$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

For a continuous distribution of charges:

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$$

Potential gradient

The components of the electric field can be found by taking partial derivatives of the electric potential:

Electric field components found from potential:

Each electric field component ...

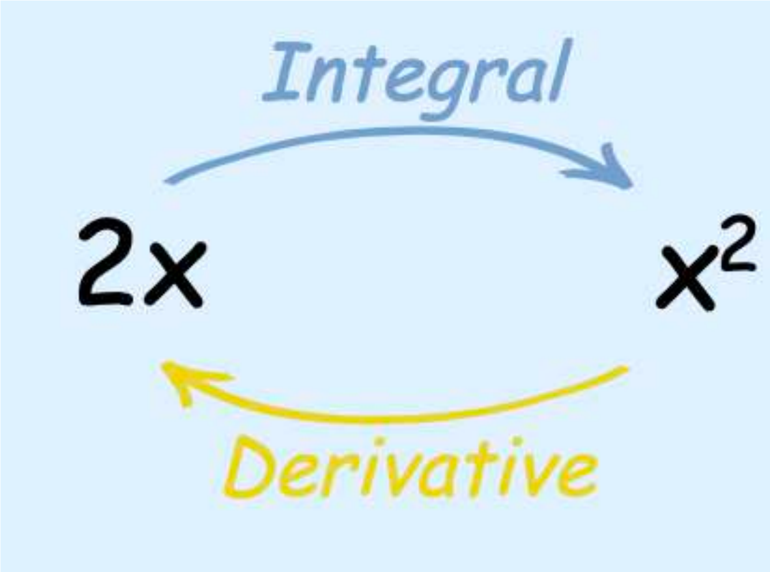
$$E_x = -\underbrace{\frac{\partial V}{\partial x}} \qquad E_y = -\underbrace{\frac{\partial V}{\partial y}} \qquad E_z = -\underbrace{\frac{\partial V}{\partial z}}$$

... equals the negative of the corresponding partial derivative of electric potential function V.

The electric field is the negative gradient of the potential:

$$\vec{E} = -\vec{\nabla} V$$

Reminder: differentiation vs integration



$\int x^n \, dx$	$\frac{x^{n+1}}{n+1} + C$

$$F \propto \frac{q_1 q_2}{r^2}$$

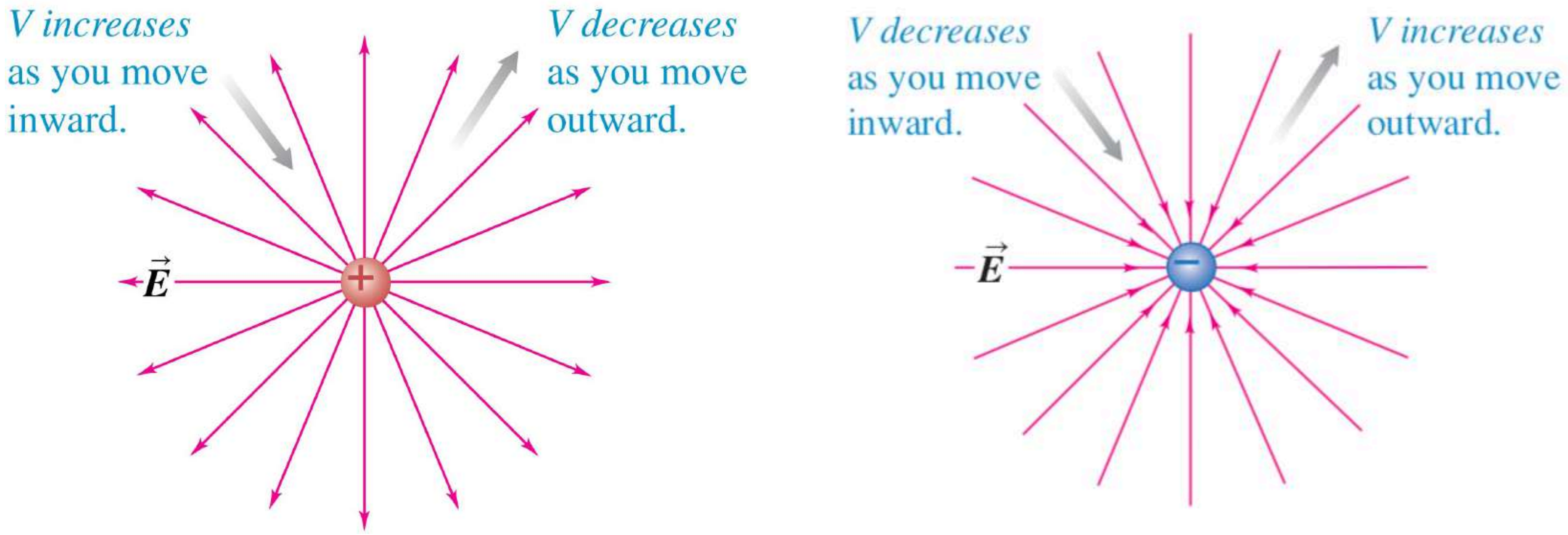
$$U \propto \int F \, dr \propto \frac{q_1 q_2}{r}$$

$$E = \frac{F}{q} \propto \frac{q}{r^2}$$

$$V \propto \int E \, dr \propto \frac{q}{r}$$

Electric potential

If you move in the direction of the electric field, the electric potential decreases, but if you move opposite the field, the potential increases.



Electric potential

r/AskPhysics

Posts

Posted by u/FallsZero 9 months ago

2

Why do we care about electric potential vs electric potential energy

Is it because electric potential is independent of the probe charge so its an easier value to use than
electric p
opportur
to go fro

8 Com

GiraffeNeckBoy 2 points · 9 months ago

An electric potential is independent of what exists within it (well, in ideal worlds where we have test charges and whatnot that don't disturb the potential), so it's handy, If you have a potential, then you can predict how anything you put in it will behave. At point x I want to put a positive test charge? Donezo. How about if I assume a cow is a point particle and she has a charge of 20 Coulombs (she's a point particle cow for christ's sake give me some creative license!) then put her in the potential? I can see how she behaves too. The potential is much much much more general than a potential energy, and is defined without needing something to act on, but no less useful, so it makes sense to use potentials. The application of the potential to something is simple extension.

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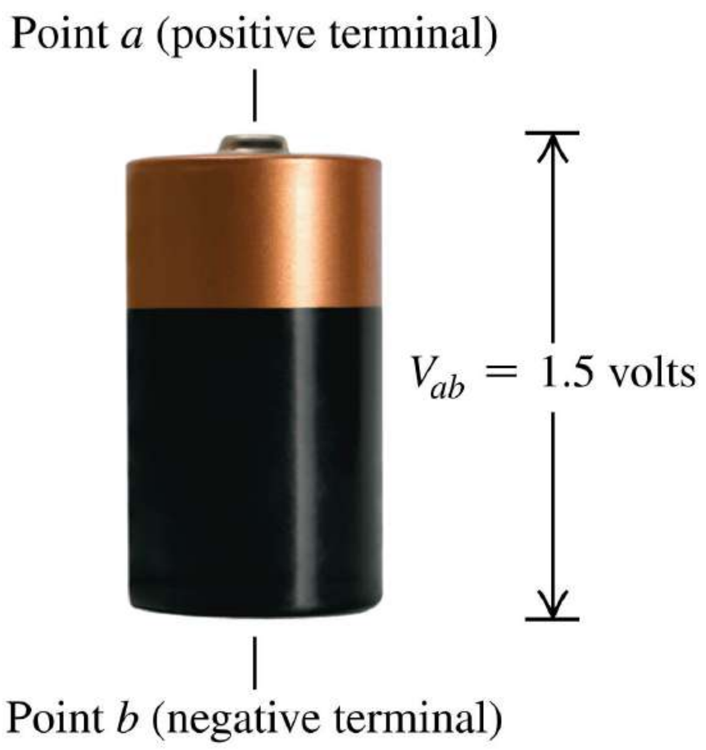
https://www.reddit.com/r/AskPhysics/comments/bhdd8k/why_do_we_care_about_electric_potential_vs/

Potential difference

The **potential difference** (e.g., $V_a - V_b$) equals the work done per unit charge to move a unit charge from one point to another (e.g., from b to a):

$$V_a - V_b = -\int_b^a \vec{E} \cdot d\vec{l}$$



In the real world: electron volts

When a particle with charge q moves from a point where the potential is V_b to a point where it is V_a, the change in the potential energy U is

$$U_a - U_b = q(V_a - V_b) = q\,V_{ab}$$

If charge q equals the magnitude e of the electron charge (1.602 x 10^{-19} C), and the potential difference V_{ab} is 1 V, the change in energy is

$$U_a - U_b = 1.602 \times 10^{-19} * 1 = 1.602 \times 10^{-19}\ \mathrm{J}$$

This quantity is defined as an electron volt:

$$1\ \mathrm{eV} = 1.602 \times 10^{-19}\ \mathrm{J}$$

What produces the most eVs?

a) Flying mosquito

b) Higgs boson

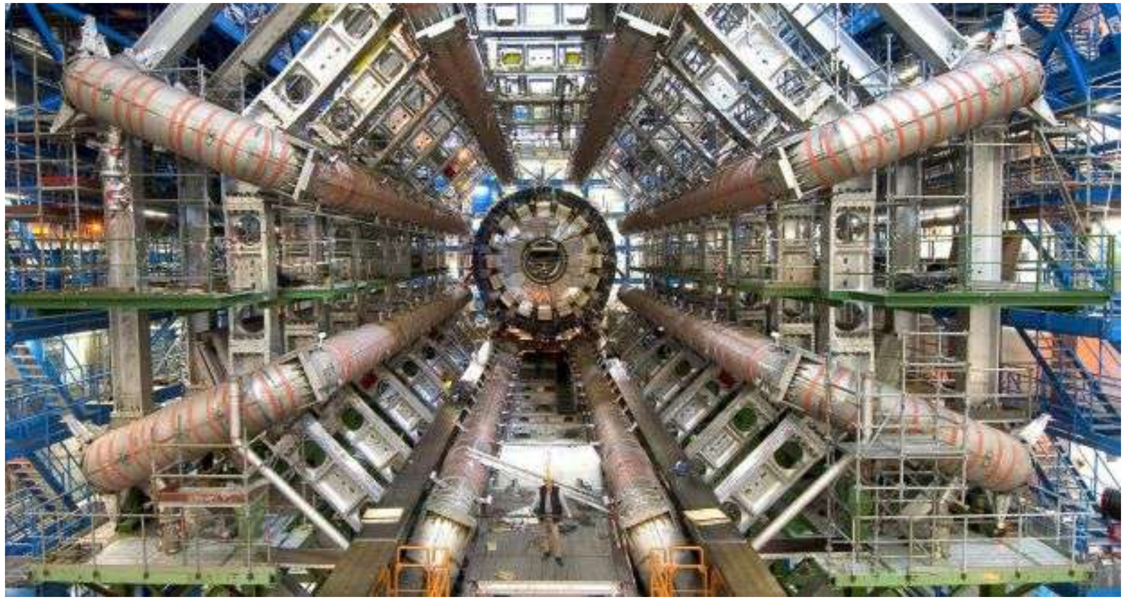
c) Visible light

d) Cosmic microwave background

e) Atomic bomb

sli.do/1e4

In the real world: electron volts



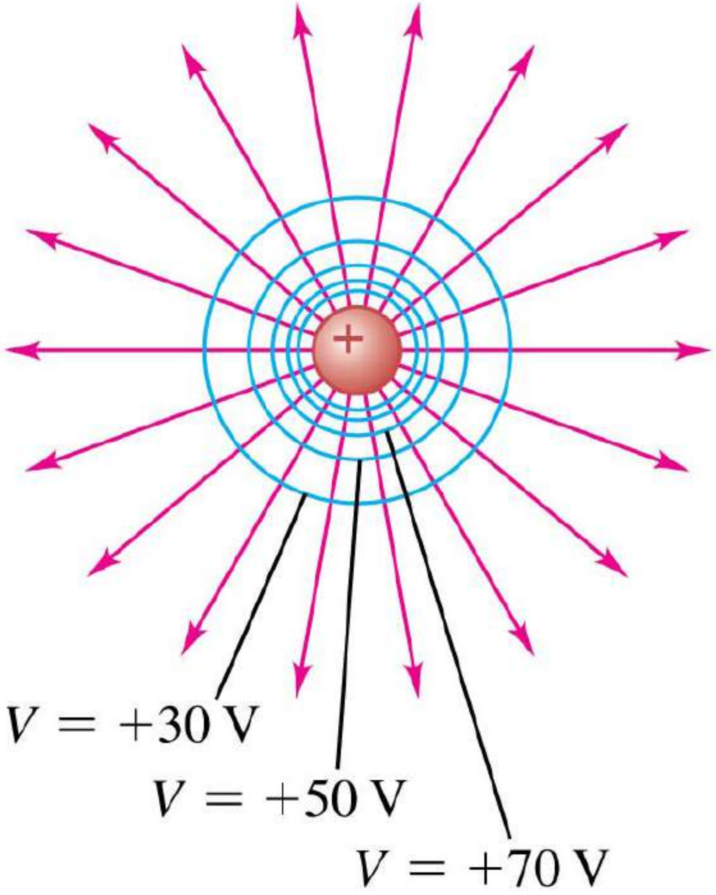
The Large Hadron Collider near Geneva, Switzerland, is designed to accelerate protons to a kinetic energy of 7 TeV.

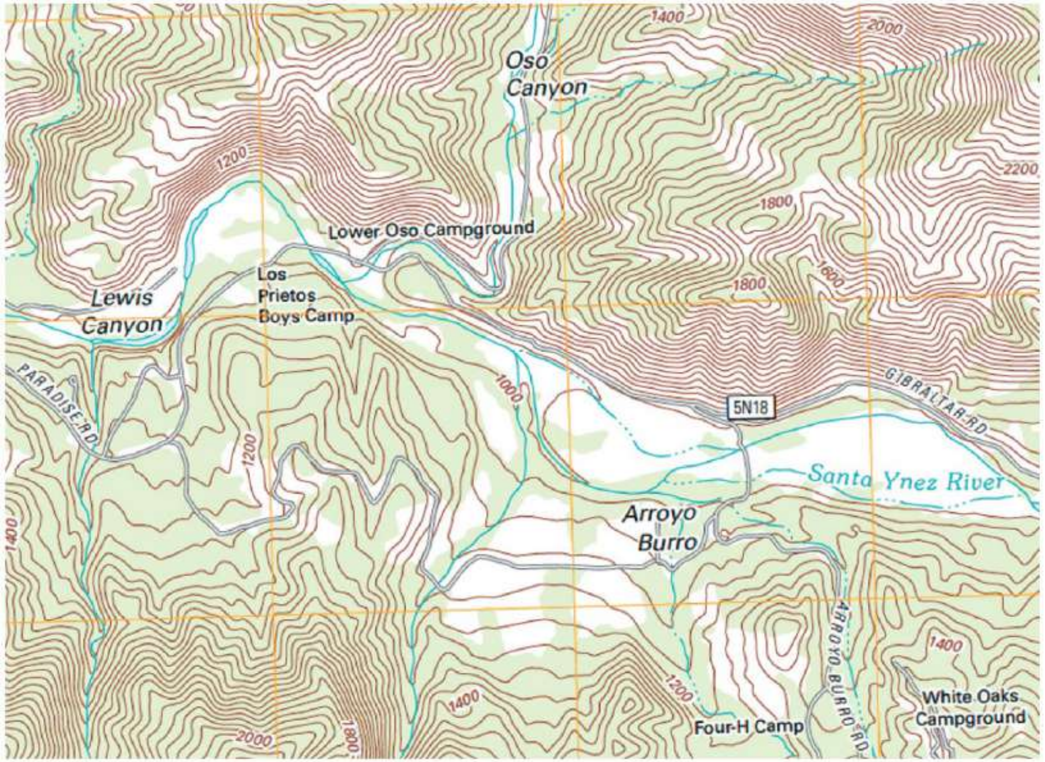
Cancer radiotherapy: each electron has a kinetic energy of 4 to 20 MeV (penetrates a few cm into a patient)

Equipotential surfaces

The actual equipotential surfaces are 3D!

An **equipotential surface** is a surface on which the electric potential is the same at every point.





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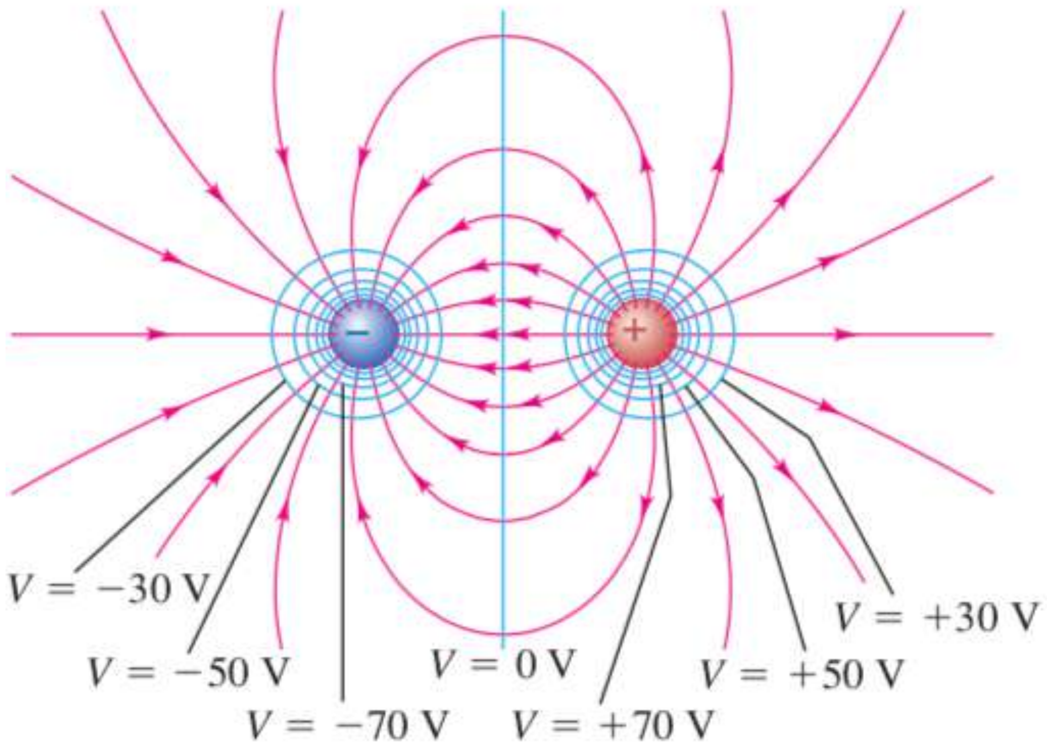
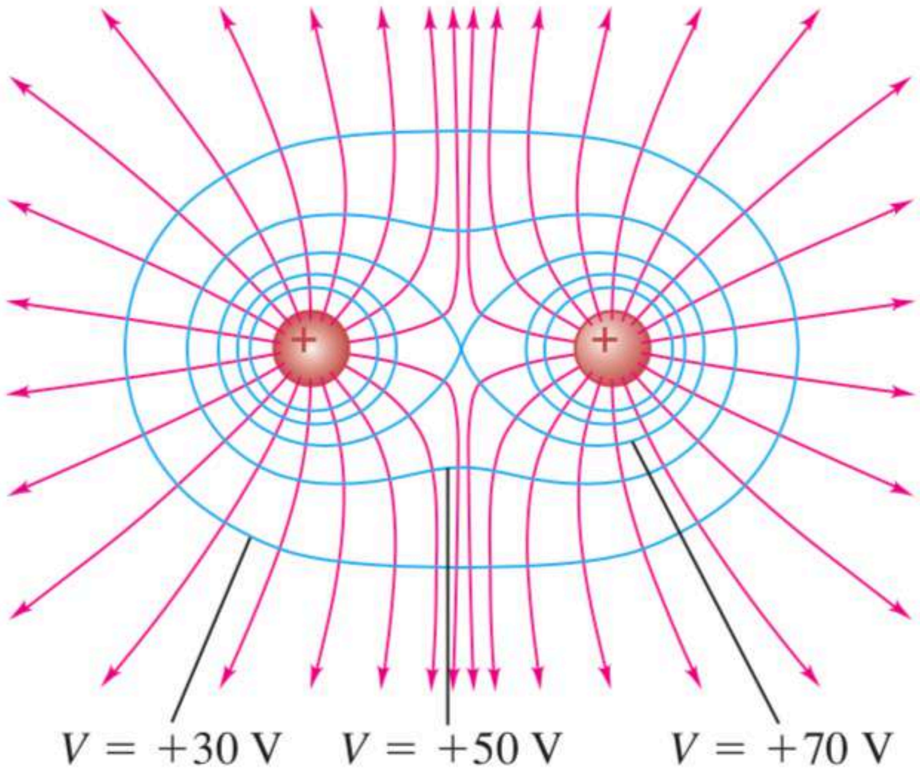
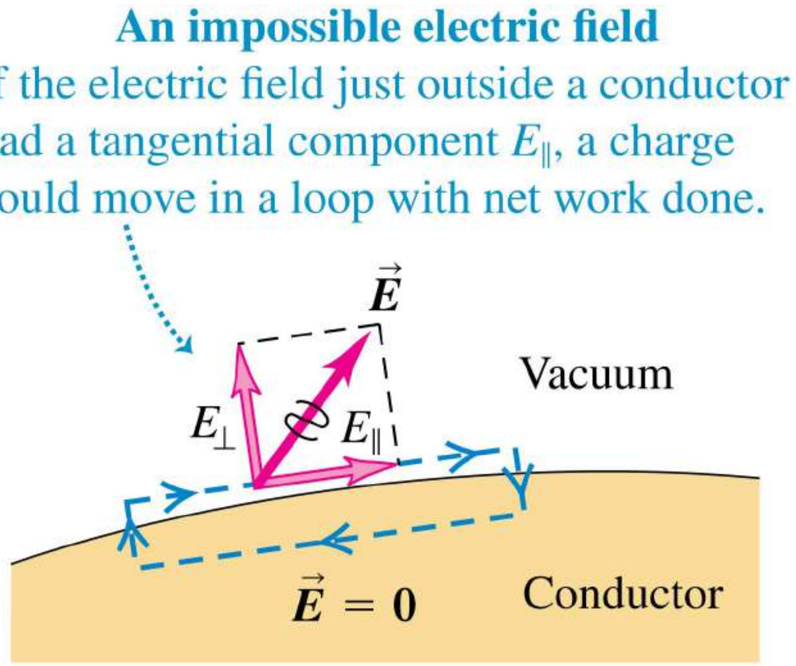
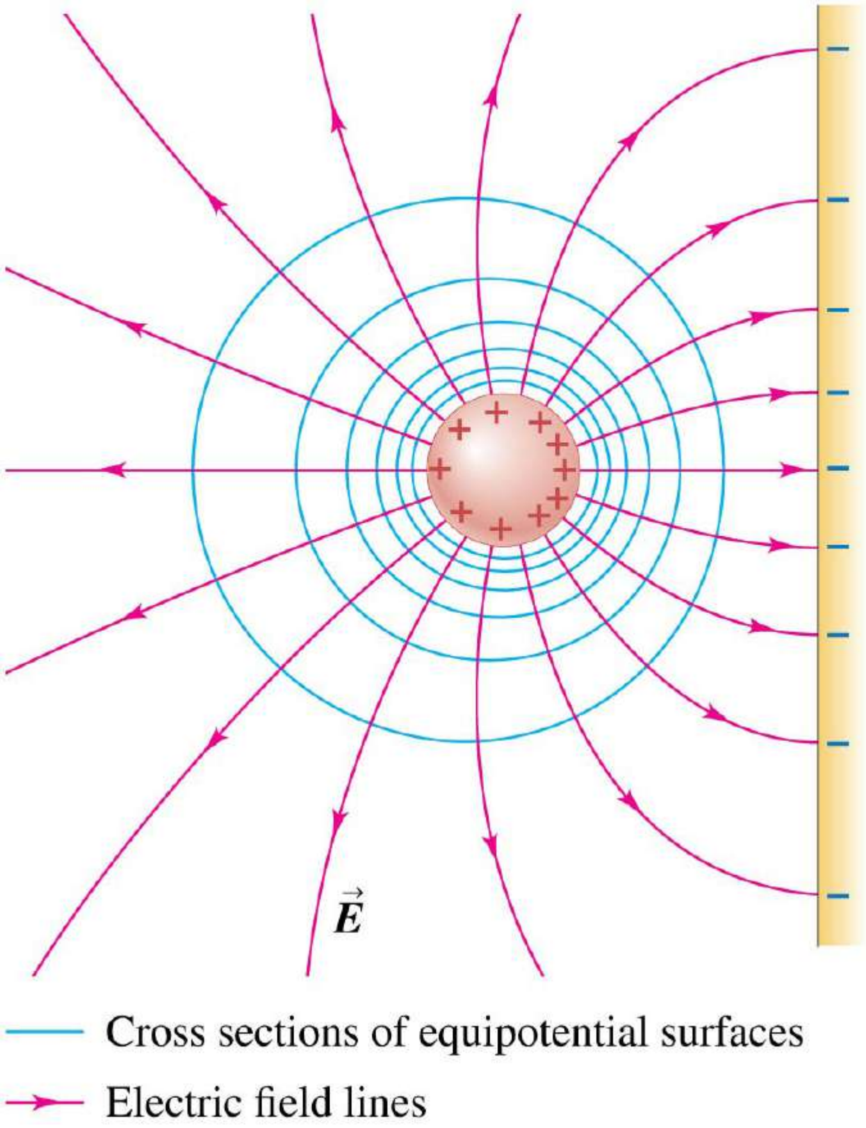




Equipotential surfaces

When all charges are at rest the surface of a conductor is always an equipotential surface.





Summary

- Gauss's Law CH 22

$$\Phi_E = E\ A \cos \emptyset$$

- Electric potential energy CH 23

$$U = q_0 V = \frac{1}{4\pi\varepsilon_0}\frac{q q_0}{r}$$

- Electric potential CH 23

Next time: Capacitance and dielectrics CH 24